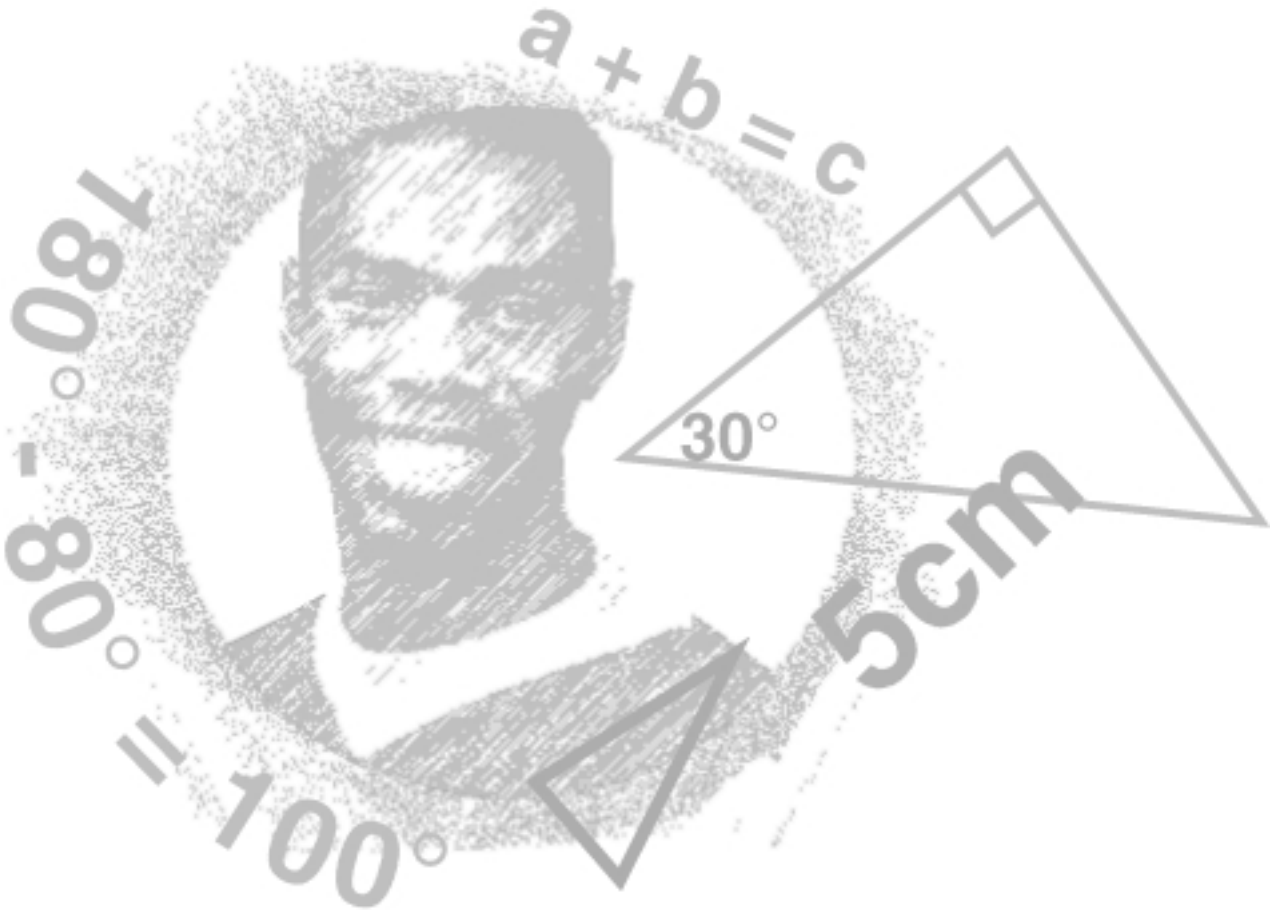
LESSON F5.3 – GEOMETRY III



Overview

You have learned how to identify different types of polygons, including triangles, and how to find their perimeter and their area.

In this lesson, you will learn more about triangles. You will study the properties of triangles in which two or three lengths or two or three angles have the same measure. You will then work with similar triangles, triangles with the same shape but different size. Also, you will study right triangles and the Pythagorean Theorem. Finally, you will learn more about parallelograms and parallel lines.

Before you begin, you may find it helpful to review the following mathematical ideas which will be used in this lesson:

To see these Review problems worked out, go to the Overview module of this lesson on the computer.

Review 1 Use supplementary angles.

$\angle ABC$ and $\angle DEF$ are supplementary angles. $m\angle ABC = 144°$. Find $m\angle DEF$.

Answer: 36°

Review 2 Use vertical angles.

$\angle XZV$ and $\angle UZV$ are vertical angles. $m\angle XZV = 44.6°$. Find $m\angle UZV$.

Answer: 44.6°

Review 3 Square a whole number.

Find the value of this expression: $7^2 + 8^2$.

Answer: 113

Review 4 Find the square root of a whole number.

Find the value of this expression: $\sqrt{6^2 + 8^2}$.

Answer: 10

Review 5 Work with a proportion.

Let's solve this problem using a proportion.

Mary finishes 3 homework problems in 5 minutes. Working at the same rate, find x, the number of problems she will finish in 30 minutes. Here's a proportion we can use to find x: $\frac{3}{5} = \frac{x}{30}$. What is the value of x?

Answer: 18

Explain

In Concept 1: Triangles and Parallelograms, you will learn about:

- **The Sum of the Angle Measures of a Triangle**
- **Congruent Triangles**
- **Isosceles Triangles and Equilateral Triangles**
- **Right Triangles and the Pythagorean Theorem**
- **Parallel Lines and Parallelograms**

CONCEPT 1: TRIANGLES AND PARALLELOGRAMS

The Sum of the Angle Measures of a Triangle

In solving problems about angles in triangles, it is useful to know the total number of degrees in the angles of a triangle.

The angle measures of a triangle add to 180 degrees.

An example is shown in Figure 1.

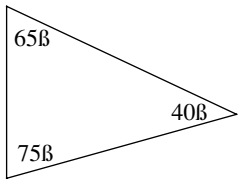


Figure 1

$40^\circ + 65^\circ + 75^\circ = 180^\circ$

You may find these Examples useful while doing the homework for this section.

Example 1

1. In Figure 2, find the missing angle measure in ΔABC.

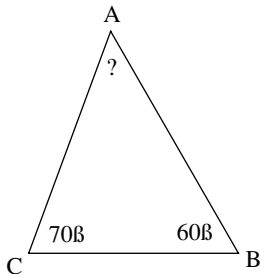


Figure 2

We can name a triangle using its 3 vertices (corners).
We can list the vertices in any order.
"ΔABC" means "the triangle with vertices A, B, and C."
"∠A" means "the angle with vertex A."

Here's one way to find the missing angle measure in this triangle:

- *Add the given angle measures.* $60^\circ + 70^\circ = 130^\circ$
- *Subtract that result from 180°.* $180^\circ - 130^\circ = 50^\circ$

So, the measure of ∠A is 50°.

2. In Figure 3, find the missing angle measure in ΔDEF. **Example 2**

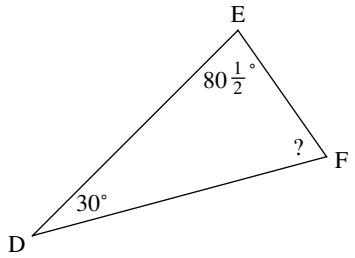


Figure 3

Here's one way to find the missing angle measure in this triangle:

- *Add the given angle measures.* $80\frac{1}{2}° + 30° = 110\frac{1}{2}°$
- *Subtract that result from 180°.* $180° - 110\frac{1}{2}° = 69\frac{1}{2}°$

So, the measure of ∠F is $69\frac{1}{2}°$.

3. In Figure 4, the measures of two angles are shown. Find the measure of ∠S. **Example 3**

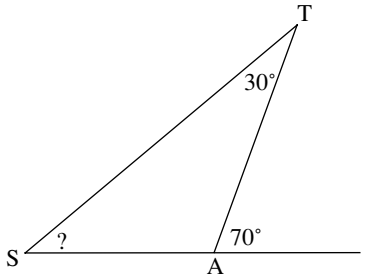


Figure 4

Here's one way to find the measure of ∠S.

The measures of supplementary angles add to 180°.	$m\angle S + m\angle T + m\angle A = 180°$
Substitute 30° for m∠T and 110° for m∠A.	$m\angle S + 30° + 110° = 180°$
Subtract 30° and 110° from both sides.	$m\angle S = 180° - 30° - 110°$
Solve for m∠S.	$m\angle S = 40°$

So, m∠S is 40°.

"≅" means "is congruent to."

"$\overline{AB}$" means "the line segment connecting vertices A and B."

Congruent Triangles

Figures with the same shape and size are called congruent figures. See Figure 5.

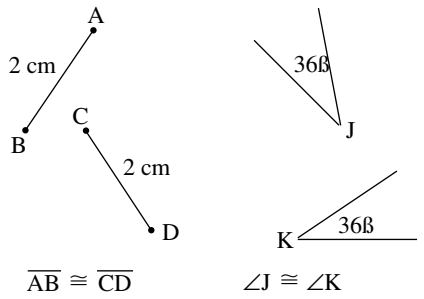


Figure 5

For two congruent triangles, we can match all 3 sides and all 3 angles.

In Figure 6, matching marks show the corresponding sides and angles of the two congruent triangles.

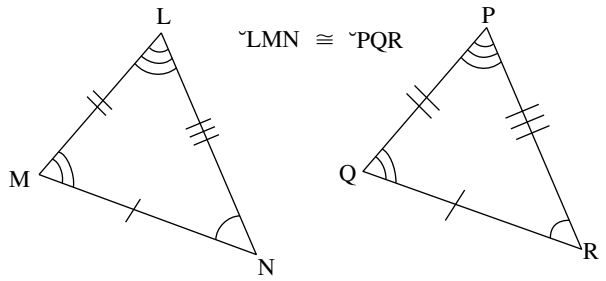


Figure 6

The order of letters in the congruence relation matches the corresponding angles or vertices of each triangle. Since ΔLMN ≅ ΔPQR,

∠L corresponds to ∠P.

∠M corresponds to ∠Q.

∠N corresponds to ∠R.

You may find these Examples useful while doing the homework for this section.

Example 4

4. In Figure 7, ΔRUN is congruent to ΔFST. Which angle of ΔFST is congruent to ∠N?

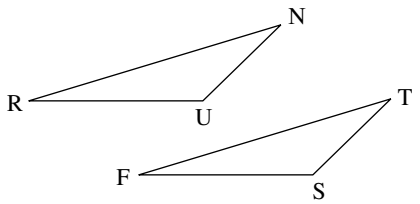
Figure 6

One way is to use the order of the vertices in the given congruence.

N is the third letter of ΔRUN.
T is the third letter of ΔFST.

So, ∠N is congruent to ∠T.

5. In Figure 7, ΔRUN ≅ ΔFST. Which side of ΔFST is congruent to $\overline{UN}$? **Example 5**



ˇRUN ≅ ˇFST

Figure 7

One way is to use the order of the vertices in the given congruence.
$\overline{UN}$ is congruent to $\overline{ST}$. *ΔRUN ≅ ΔFST*

Here are the other pairs of congruent sides:
$\overline{RU}$ is congruent to $\overline{FS}$. *ΔRUN ≅ ΔFST*
$\overline{RN}$ is congruent to $\overline{FT}$. *ΔRUN ≅ ΔFST*

6. The triangles in Figure 8 are congruent. Write a congruence relation for these triangles. **Example 6**

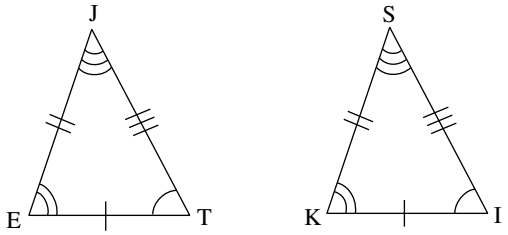


Figure 8

Here's one way to write a congruence relation:
- *First, write one pair of corresponding vertices.* *ΔJ* *ΔS*
- *Then, write another pair of corresponding vertices.* *ΔJE* *ΔSK*
- *Finally, write the last pair of corresponding vertices.* *ΔJET* ≅ *ΔSKI*

So, one congruence relation for these triangles is ΔJET ≅ ΔSKI.

Any congruence relation that matches corresponding vertices is correct. Here are 3 more examples:

ΔETJ ≅ ΔKIS
ΔJTE ≅ ΔSIK
ΔTEJ ≅ ΔIKS

Example 7

7. The triangles in Figure 9 are congruent. Find the measure of ∠B.

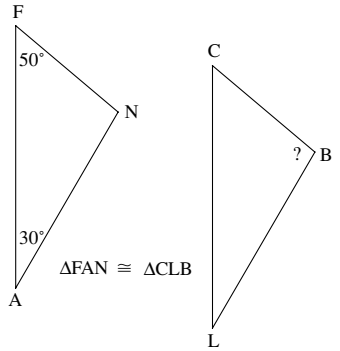


Figure 9

The congruence relation states that ∠B corresponds to ∠N.
So, ∠B and ∠N have the same measure.

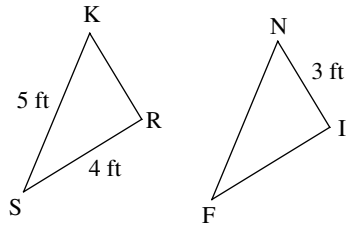
Here's one way to find the measure of ∠N.

Add 50° and 30°.	*50° + 30° = 80°*
Subtract that result from 180°.	*180° – 80° = 100°*

So, ∠B and ∠N each have measure 100°.

Example 8

8. The triangles in Figure 10 are congruent. Find the length of $\overline{RK}$.



ΔSRK ≅ ΔFIN

Figure 10

The congruence relation states that $\overline{RK}$ corresponds to $\overline{IN}$.
So, $\overline{RK}$ and $\overline{IN}$ have the same length.
The length of $\overline{IN}$ is 3 ft.
So, the length of $\overline{RK}$ is also 3 ft.

Isosceles Triangles and Equilateral Triangles

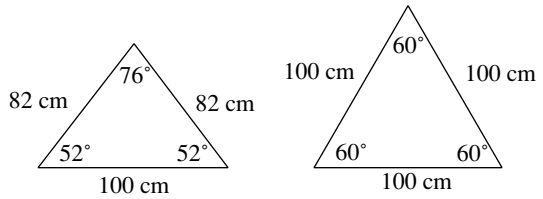
There are several special types of triangles. An Isosceles triangle is one such type.

An isosceles triangle has at least two sides that have the same length. In Figure 11, each triangle is an isosceles triangle.

*The word **isosceles** comes from two Greek words:*

ISOS	+	***SKELOS***
"equal"		*"leg"*

So isosceles means having two or three "equal legs."



Isosceles Triangles

Figure 11

In an isosceles triangle, the angles that are opposite the equal-length sides have the same measure.

If a triangle has two angles with equal measures, then the opposite sides also have equal lengths, and the triangle is an isosceles triangle.

Equilateral Triangles

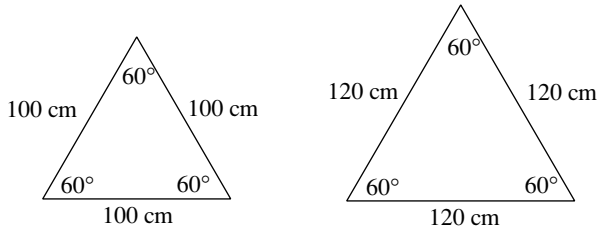
An Equilateral triangle is another special type of triangle.

An equilateral triangle has three sides of equal length. In Figure 12, each triangle is an equilateral triangle.

*In the word **equilateral**, the prefix "equi" means "equal" and "lateral" means "side."*

EQUI	+	***LATERAL***
"equal"		*"side"*

So equilateral means having three "equal sides."



Equilateral Triangles

Figure 12

In an equilateral triangle, all three angle measures are equal. Since the angle measures in a triangle add to 180°, each angle in an equilateral triangle measures 180° ÷ 3. So, the measure of each angle is 60°.

If each angle of a triangle measures 60°, then the triangle is an equilateral triangle.

You may find these Examples useful while doing the homework for this section.

Example 9

9. In Figure 13, ΔPAT is an isosceles triangle. Which pair of angles **must** have the same measure?

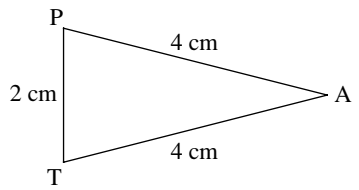


Figure 13

In ΔPAT, $\overline{PA}$ is the same length as $\overline{AT}$.
So the angles opposite these sides have the same measure.
That is, m∠T = m∠P.

Example 10

10. In Figure 14, ΔTOP is an isosceles triangle. Find the measure of ∠P.

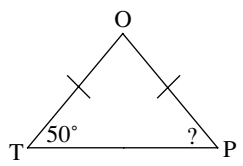


Figure 14

In an isosceles triangle, the angles across from the equal-length sides have equal measure. The matching marks on $\overline{TO}$ and $\overline{OP}$ indicate that these sides have equal lengths.

So, ∠T and ∠P have the same measure, 50°.

Example 11

11. Find the length of $\overline{SN}$ in ΔSUN, as shown in Figure 15.

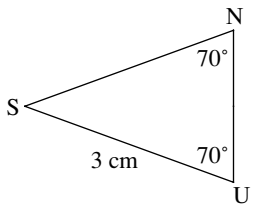


Figure 15

Since two angles, ∠N and ∠U, of ΔSUN have equal measures, the triangle is isosceles.
In an isosceles triangle, the sides opposite equal-measure angles have the same length.

So $\overline{SN}$ has the same length as $\overline{SU}$, 3 cm.

12. Sketch an isosceles triangle ΔSAW, with $\overline{SA}$ and $\overline{SW}$ of equal lengths. **Example 12**
Figure 16 is one possible answer.

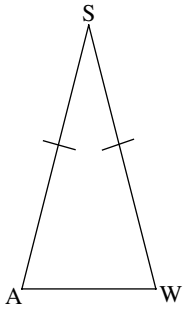


Figure 16

13. In Figure 17, ΔFAX is an equilateral triangle. Find the length of $\overline{FA}$. **Example 13**

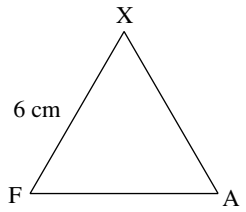


Figure 17

In an equilateral triangle, all 3 sides have the same length.
So $\overline{FA}$ has the same length as $\overline{FX}$, 6 cm. ($\overline{AX}$ also has length 6 cm.)

14. In the triangle in Figure 18, find the measure of angle Q. **Example 14**

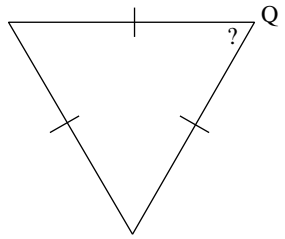


Figure 18

Since all 3 sides have matching marks, the 3 sides have equal lengths.
That means the triangle is an equilateral triangle.
All 3 angles have the same measure. $180° \div 3 = 60°$

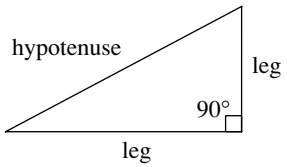
So, $m\angle Q = 60°$.

"$m\angle Q$" means the measure of angle Q.

Right Triangles and the Pythagorean Theorem

Another special type of triangle is a right triangle.

A right triangle has one right angle that measures 90˚, as shown in Figure 19.



Right Triangle

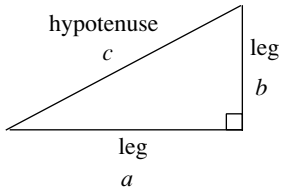
Figure 19

In a right triangle, the two sides that form the right angle are called the legs of the triangle. The side opposite the right angle is called the hypotenuse. It's the longest side.
In a right triangle, the right angle is the largest angle. The other two angles are acute angles.

The Pythagorean Theorem

The Pythagorean Theorem describes an important relationship among the sides of a right triangle, as shown in Figure 21.

Pythagorean Theorem



$$a^2 + b^2 = c^2$$

Figure 21

a, b, and c are the lengths of the sides, as shown in Figure 21.

In a right triangle, the sum of the squares of the legs equals the square of the hypotenuse: $a^2 + b^2 = c^2$

The Pythagorean Theorem is true only for right triangles. This gives us a way to test whether a triangle is a right triangle. See Figure 22.

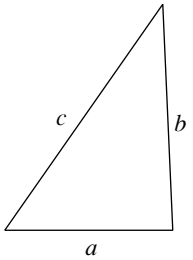
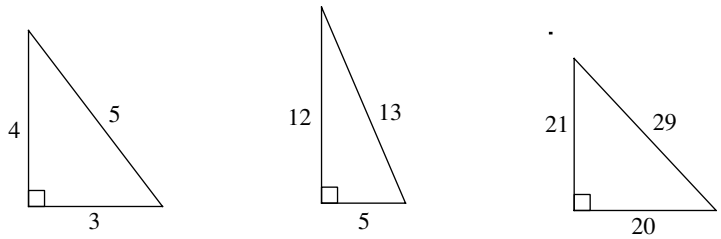


Figure 22

- If $a^2 + b^2 = c^2$, then the triangle **is** a right triangle.
- If $a^2 + b^2 \neq c^2$, then the triangle **is not** a right triangle.

(Here, a, b, and c are the lengths of the sides. The length of the longest side, the hypotenuse, is c.)

A Pythagorean triple is a group of three whole numbers that can be used as the lengths of the sides of a right triangle. Figure 23 shows some examples of Pythagorean triples.



(3, 4, 5)	(5, 12, 13)	(20, 21, 29)
(6, 8, 10)	(10, 24, 26)	(40, 42, 58)
(9, 12, 15)	(15, 36, 39)	(60, 63, 87)
(12, 16, 20)	(20, 48, 52)	(80, 84, 116)

Figure 23

Example 15

You may find these Examples useful while doing the homework for this section.

15. In a given right triangle, the measure of one angle is 35°. Find the measures of the other angles.

Figure 24 shows a right triangle with one angle whose measure is 35°.

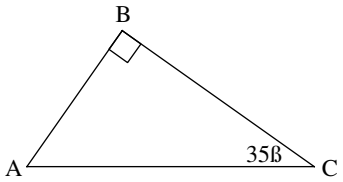


Figure 24

One of the angles in a right triangle must be a right angle. It's labeled ∠B.
Since ∠B is a right angle, m∠B = 90°.
To find the measure of ∠A, add 90° and 35°. $90° + 35° = 125°$
Subtract that result from 180°. $180° - 125° = 55°$

So, m∠A = 55°.

Example 16

16. In a certain right triangle, the lengths of the sides are 9 cm, 12 cm, and 15 cm. Which of these lengths is the length of the hypotenuse?

The hypotenuse is the longest side, so the length of the hypotenuse is 15 cm, as shown in Figure 25.

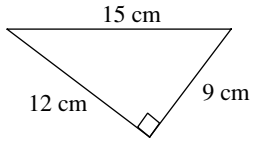


Figure 25

Example 17

17. In ΔTRI, the lengths of the sides are 4 ft, 5 ft, and 6 ft. Is ΔTRI a right triangle? See Figure 26.

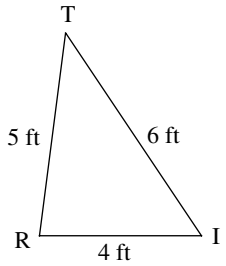


Figure 26

Here's one way to find out whether ΔTRI is a right triangle:

• *Square the length of each side.*	$4^2 = 4 \times 4 = 16$
	$5^2 = 5 \times 5 = 25$
	$6^2 = 6 \times 6 = 36$
• *Add the two smaller squares and see if the result equals the largest square.*	*Is 16 + 25 = 36?*
	Is 41 = 36? No.

Since 16 + 25 = 41 (not 36), ΔTRI is ***not*** *a right triangle.*

Example 18

18. In Figure 27, the triangle shown is a right triangle. Find the length of its hypotenuse.

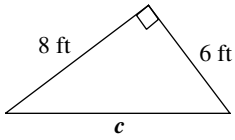


Figure 27

Here's one way to find the length of the hypotenuse.

Use the Pythagorean Theorem.	$a^2 + b^2 = c^2$
Square the given lengths.	$6^2 + 8^2 = c^2$
Add the squares.	$36 + 64 = c^2$
	$100 = c^2$
To find c, take the square root of 100.	$10 = c$

The length of the hypotenuse is 10 ft.

Example 19

19. In Figure 28, the triangle shown is a right triangle. Find *a*, the length of one of its legs.

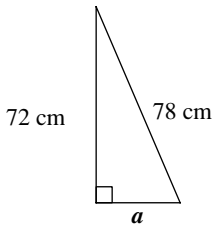


Figure 28

Here's one way to find the length of the leg.

Use the Pythagorean Theorem.	a^2	+	b^2	=	c^2
Square the given lengths.	a^2	+	72^2	=	78^2
Add the squares.	a^2	+	5184	=	6084
To get a^2 by itself, subtract 5184			$-\ 5184$		-5184
from both sides.	a^2			=	900
To find a, take the square root of 900.	a			=	30

The length of the leg is 30 cm.

Parallel Lines and Parallelograms

You have learned about parallel lines. Now you will study some properties of the angles formed by a line that crosses two parallel lines.

In Figure 29, line ***k*** and line ***n*** are parallel. Line ***t*** crosses lines ***k*** and ***n***, and forms eight angles, which are shown.

*When line **k** is parallel to line **n**, we write **k** // **n**.*

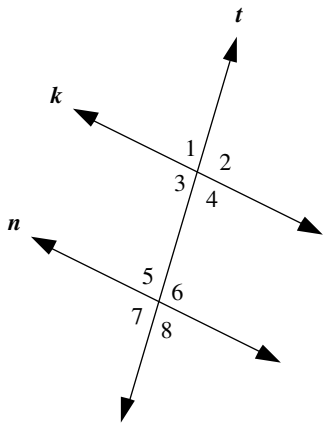


Figure 29

As shown in Figure 30, there are 4 pairs of vertical angles.

Vertical angles are opposite angles.

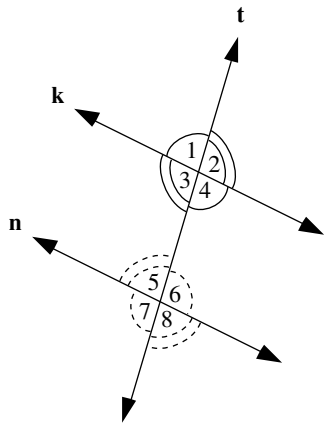


Figure 30

∠1 and ∠4	are vertical angles
∠2 and ∠3	are vertical angles
∠5 and ∠8	are vertical angles
∠6 and ∠7	are vertical angles

*Vertical angles have the same measure whether lines **k** and **n** are parallel or not.*

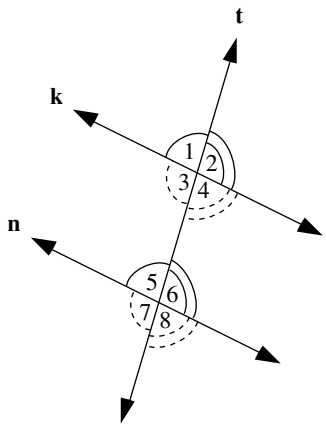
Vertical Angles

Vertical angles have the same measure.

m∠1 = m∠4	m∠2 = m∠3
m∠5 = m∠8	m∠6 = m∠7

Corresponding angles are two angles on the same side of line ***t*** that are both "above" or both "below" lines ***k*** and ***n***.

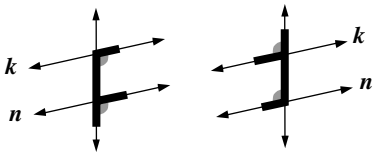
There are 4 pairs of corresponding angles, as shown in Figure 31.



∠1 and ∠5	are corresponding angles
∠2 and ∠6	are corresponding angles
∠3 and ∠7	are corresponding angles
∠4 and ∠8	are corresponding angles

Figure 31

Corresponding angles form an "F." This "F" may face in any direction.



Caution! *Line **k** and line **n** must be parallel for corresponding angles to have the same measure.*

Corresponding Angles

Because line ***k*** ∥ line ***n***, corresponding angles have the same measure.

m∠1 = m∠5	m∠2 = m∠6
m∠3 = m∠7	m∠4 = m∠8

Alternate interior angles are two angles on opposite sides of line ***t*** that are both "inside" line ***k*** and line ***n***.

There are 2 pairs of alternate interior angles, as shown in Figure 32.

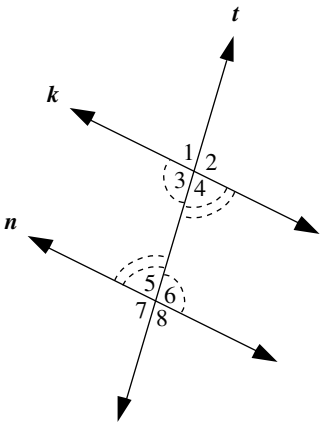
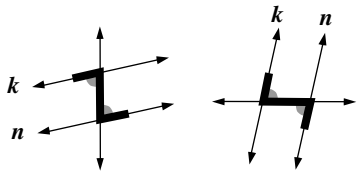


Figure 32

∠3 and ∠6	are alternate interior angles
∠4 and ∠5	are alternate interior angles

Alternate interior angles form a "Z." This "Z" may face in any direction.



Caution! *Lines **k** and **n** must be parallel for alternate interior angles to have the same measure.*

Alternate Interior Angles

Because line ***k*** ∥ line ***n***, alternate interior angles have the same measure.

m∠3 = m∠6 m∠4 = m∠5

Parallelograms

Recall that a parallelogram is a polygon with 4 sides that has two pairs of parallel sides. See Figure 33. In a parallelogram, the opposite sides have the same length.

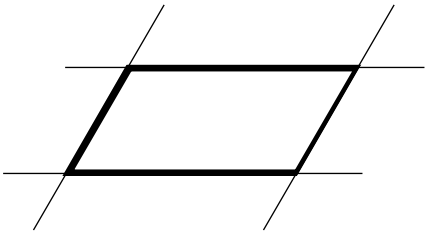


Figure 33

In Figure 34, all the marked angles with "*(*" or "*)*" have the same measure as angle 1. That's because vertical angles have the same measure and corresponding angles formed by parallel lines have the same measure.

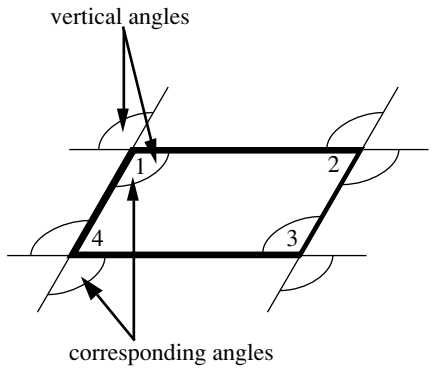


Figure 34

Notice that angle 1 and angle 3 have the same measure. In the same way, angle 2 and angle 4 have the same measure. It follows that:

Opposite angles of a parallelogram have the same measure.

In Figure 35, since opposite sides of the parallelogram are parallel, the corresponding angles have the same measure. The measures of supplementary angles add to 180°.

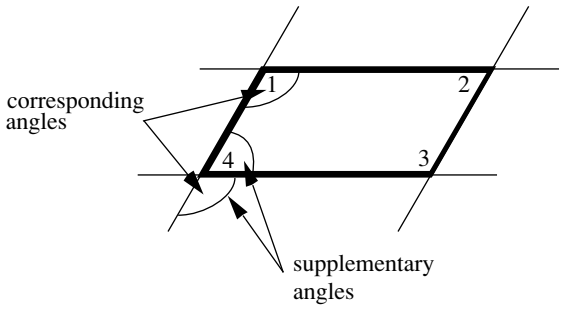


Figure 35

It follows that:

Consecutive angles of a parallelogram are supplementary angles.

20. In Figure 36, $j \parallel u$. The measure of angle 1 is 80°. Find the measures of the other angles.

Example 20

You may find these Examples useful while doing the homework for this section.

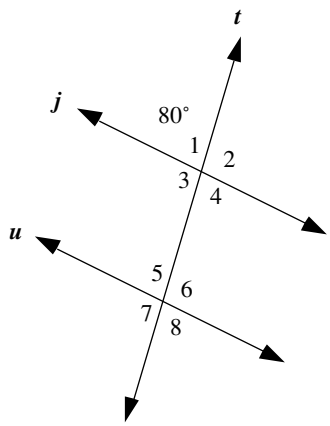


Figure 36

Here's one way to find the measures of the other angles.

∠4 and ∠1 are vertical angles, so they have the same measure.	$m\angle 4 = m\angle 1 = 80°$
∠2 and ∠1 are supplementary angles, so their measures add to 180°. 180° – 80° = 100°	$m\angle 2 = 100°$
∠3 and ∠2 are vertical angles, so they have the same measure.	$m\angle 3 = m\angle 2 = 100°$
∠5 and ∠1 are corresponding angles, so they have the same measure.	$m\angle 5 = m\angle 1 = 80°$
∠6 and ∠2 are corresponding angles, so they have the same measure.	$m\angle 6 = m\angle 2 = 100°$
∠7 and ∠3 are corresponding angles, so they have the same measure.	$m\angle 7 = m\angle 3 = 100°$
∠8 and ∠4 are corresponding angles, so they have the same measure.	$m\angle 8 = m\angle 4 = 80°$

Here's another way to find m∠6: Since ∠6 and ∠3 are alternate interior angles, m∠6 = m∠3 = 100°.

Similarly, since ∠4 and ∠5 are alternate interior angles, m∠4 = m∠5 = 80°.

Example 21

21. Line segment $\overline{BE}$ is parallel to line segment $\overline{CD}$. Find the missing angle measures in ΔACD and ΔABE. See Figure 37.

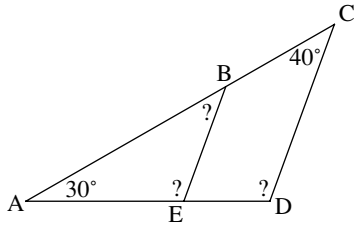


Figure 37

Here's a way to find the measure of angle ABE. Since $\overline{BE} \parallel \overline{CD}$, $\angle ABE$ and $\angle BCD$ are corresponding angles. Corresponding angles have the same measure. So, $m\angle ABE = 40^\circ$.

Here's a way to find the measure of angle AEB. The angle measures of a triangle add to 180°. Since $30^\circ + 40^\circ = 70^\circ$ and $180^\circ - 70^\circ = 110^\circ$, $m\angle AEB = 110^\circ$.

Here's a way to find the measure of angle D. Since $\overline{BE} \parallel \overline{CD}$, $\angle D$ and $\angle AEB$ are corresponding angles. Corresponding angles have the same measure. So, $m\angle D = 110^\circ$.

Example 22

22. Find the missing angle measures in parallelogram GRAM. See Figure 38.

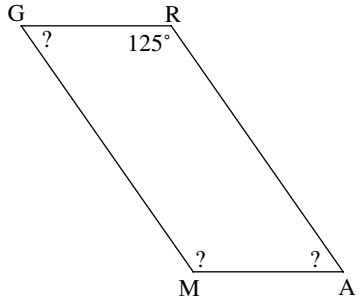


Figure 38

In a parallelogram, opposite angles have the same measure. Since angle M and angle R are opposite angles, $m\angle M = 125^\circ$.

In a parallelogram, consecutive angles are supplementary. Angle G and angle R are consecutive angles. Since $180^\circ - 125^\circ = 55^\circ$, $m\angle G = 55^\circ$.
Since angle A and angle G are opposite angles, the measure of angle A is also 55°.

Example 23

23. Find the missing side lengths in parallelogram RUSH. See Figure 39.

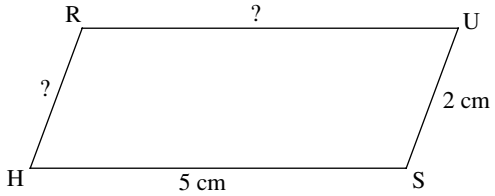


Figure 39

In a parallelogram, opposite sides have the same length.
Since $\overline{US}$ and $\overline{RH}$ are opposite sides, the length of $\overline{RH}$ is 2 cm.
Since $\overline{SH}$ and $\overline{RU}$ are opposite sides, the length of $\overline{RU}$ is 5 cm.

Explain

In Concept 2: Similar Polygons, you will find a section on each of the following:

- **Recognizing Similar Polygons**
- **Writing a Similarity Statement**
- **Using Shortcuts to Recognize Similar Triangles**
- **Finding the Measures of Corresponding Angles of Similar Triangles**
- **Finding the Lengths of Corresponding Sides of Similar Triangles**

Congruent objects must be similar. But ***not*** *all similar objects are congruent.*

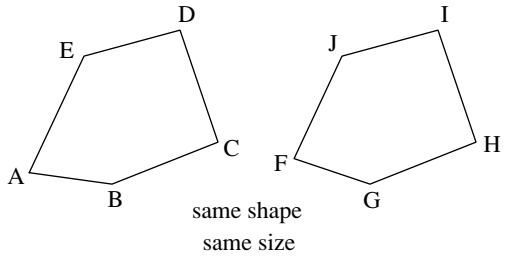
"~" means "is similar to"

CONCEPT 2: SIMILAR POLYGONS

Recognizing Similar Polygons

You have learned that congruent objects have the same shape and the same size. Similar objects must also have the same shape, but they may or may not be the same size. An architect's scale model is similar to the full-size building. The street map of a city is similar to the actual streets and blocks of the city. If you shrink a picture when you photocopy it, the two images are similar. If you enlarge a photograph, the original and the enlarged version are similar.

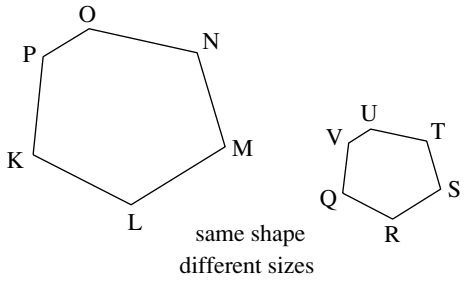
In Figure 40, the pentagons are similar polygons. They are also congruent.



ABCDE ~ FGHIJ

Figure 40

In Figure 41, the hexagons are similar polygons, but they are **not** congruent.



KLMNOP ~ QRSTUV

Figure 41

For two polygons to be similar, both of the following must be true:

- Corresponding angles have the same measure.
- Lengths of corresponding sides have the same ratio.

In Figure 42, the quadrilaterals STAR and FIND are similar, since they satisfy both of the conditions above.

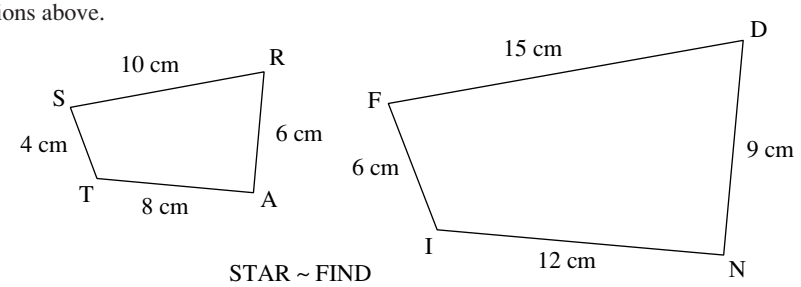


Figure 42

ST is the length of line segment $\overline{ST}$. FI is the length of line segment $\overline{FI}$, and so on.

We also say that the lengths of corresponding sides are proportional.

Corresponding angles have the same measure:

$m\angle S = m\angle F \qquad m\angle T = m\angle I$

$m\angle A = m\angle N \qquad m\angle R = m\angle D$

Corresponding lengths have the same ratio:

$\frac{ST}{FI} = \frac{4}{6} = \frac{2}{3} \qquad \frac{TA}{IN} = \frac{8}{12} = \frac{2}{3}$

$\frac{AR}{ND} = \frac{6}{9} = \frac{2}{3} \qquad \frac{RS}{DF} = \frac{10}{15} = \frac{2}{3}$

You may find these Examples useful while doing the homework for this section.

Example 24

24. Are the polygons ABCD and EFGH in Figure 43 similar?

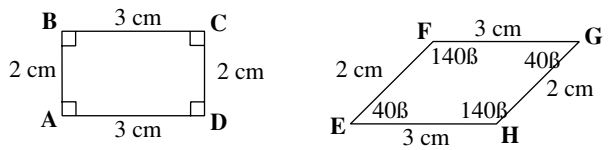


Figure 43

*The polygons are **not** similar. Corresponding lengths have the same ratio, but corresponding angles do not have the same measure:*

- $\frac{2}{2} = 1 \qquad \frac{3}{3} = 1 \qquad \frac{2}{2} = 1 \qquad \frac{3}{3} = 1$
- *All the angles in the rectangle measure 90˚.*

  Two of the angles in the parallelogram measure 40˚, and two of the angles measure 140˚.

Example 25

25. Are the polygons ABCD and EFGH in Figure 44 similar?

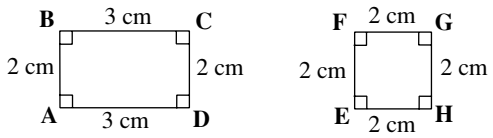


Figure 44

*The polygons in Figure 44 are **not** similar. Corresponding angles have the same measure, but lengths of corresponding sides do not have the same ratio:*

- *All the angles have the same measure, 90°.*
- $\frac{2}{2} = 1 \qquad \frac{3}{2} = 1.5 \qquad \frac{2}{2} = 1 \qquad \frac{3}{2} = 1.5$

Some lengths of corresponding sides have ratio 1 and some have ratio 1.5.

Example 26

26. In Figure 45, triangle SUM is similar to triangle CAR. Find the missing angle measures.

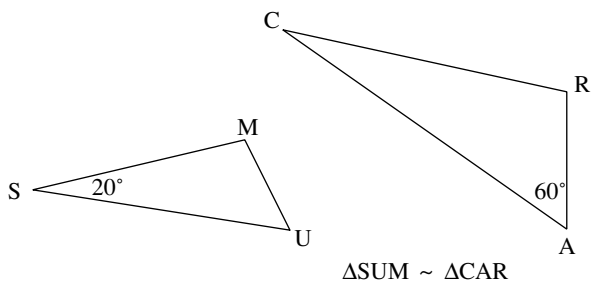


Figure 45

Corresponding angles have the same measure.
So, m∠C = m∠S = 20° and m∠U = m∠A = 60°.
You can use the fact that the angle measures in a triangle add to 180° to find the measure of the third angle in each triangle.

- *Add the angle measures you know.* $20° + 60° = 80°$
- *Subtract that result from 180°.* $180° - 80° = 100°$

So, m∠M = m∠R = 100°.

Example 27

27. In Figure 46, ΔSUM is similar to ΔCAR. Find this ratio: $\frac{\text{length of } \overline{CA}}{\text{length of } \overline{SU}}$

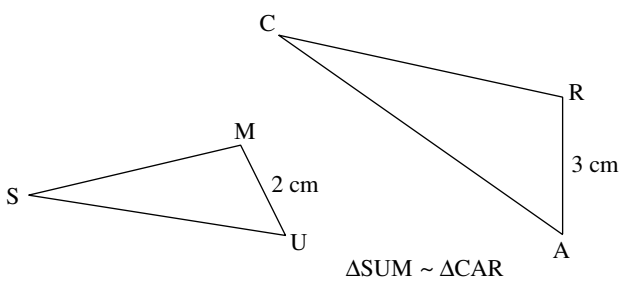


Figure 46

In similar triangles, corresponding lengths have the same ratio.

$$\frac{\text{length of } \overline{CA}}{\text{length of } \overline{SU}} = \frac{\text{length of } \overline{AR}}{\text{length of } \overline{UM}} = \frac{3}{2}$$

Writing a Similarity Statement

In Figure 47, triangle FAN and triangle CLB are similar.

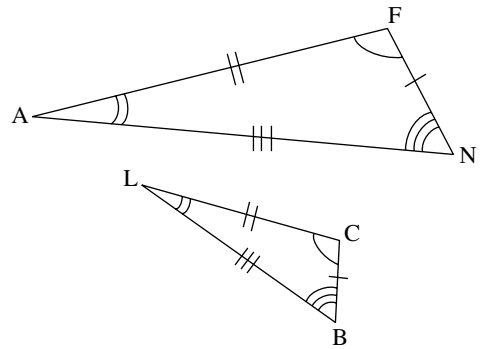


Figure 47

The order of letters in the similarity statement ΔFAN ~ ΔCLB matches the corresponding angles or vertices of each polygon.

*∠**F** corresponds to ∠**C**.*	*Δ**F**AN ~ Δ**C**LB*
*∠**A** corresponds to ∠**L**.*	*ΔF**A**N ~ ΔC**L**B*
*∠**N** corresponds to ∠**B**.*	*ΔFA**N** ~ ΔCL**B***

The order of the letters in the similarity statement also matches corresponding sides.

$\overline{FA}$ corresponds to $\overline{CL}$.	*Δ**F**A**N** ~ Δ**C**L**B***
$\overline{AN}$ corresponds to $\overline{LB}$.	*ΔF**AN** ~ ΔC**LB***
$\overline{FN}$ corresponds to $\overline{CB}$.	*Δ**FAN** ~ Δ**CLB***

You may find these Examples useful while doing the homework for this section.

Example 28

28. Write a similarity statement for the triangles in Figure 48.

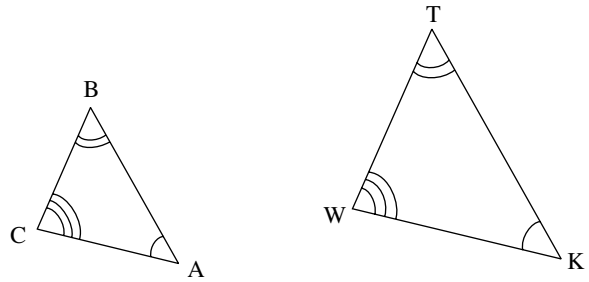


Figure 48

Here's one way to write a similarity statement:

• *First, write one pair of corresponding vertices.*	*ΔA*		*ΔK*
• *Then, write another pair of corresponding vertices.*	*ΔAB*		*ΔKT*
• *Finally, write the last pair of corresponding vertices.*	*ΔABC*	~	*ΔKTW*

So, one similarity statement for these triangles is ΔABC ~ ΔKTW.
Here are other similarity statements:

ΔBAC ~ ΔTKW
ΔACB ~ ΔKWT
ΔCAB ~ ΔWKT

29. ΔSLY ~ ΔDOG

Example 29

In ΔDOG, find the angle that has the same measure as angle Y.

Use the order of the letters in the similarity statement.
∠Y corresponds to ∠G. $\Delta SL\boldsymbol{Y} \sim \Delta DO\boldsymbol{G}$

So, angle G has the same measure as angle Y.

30. ΔSLY ~ ΔDOG

Example 30

The ratio of SL to DO is 4 to 3. Find the ratio of LY to OG.

SL is the length of line segment $\overline{SL}$. LY is the length of line segment $\overline{LY}$, and so on.

Use the order of the letters in the similarity statement.
$\overline{SL}$ corresponds to $\overline{DO}$. $\Delta \boldsymbol{SL}Y \sim \Delta \boldsymbol{DO}G$
$\overline{LY}$ corresponds to $\overline{OG}$. $\Delta S\boldsymbol{LY} \sim \Delta D\boldsymbol{OG}$

The lengths of corresponding sides have the same ratio. So the ratio of LY to OG is also 4 to 3.

Shortcuts for Determining Similar Triangles

You know that two polygons are similar if all their corresponding angles have the same measure **and** all their corresponding lengths have the same ratio.

However, you can show that two **triangles** are similar by using less information. Here are two shortcuts.

Shortcut #1

Two triangles are similar if two angles of one triangle have the same measure as two angles of the other triangle.

This shortcut is sometimes called "Angle Angle" or "AA."

Shortcut #2

Two triangles are similar if all their corresponding lengths have the same ratio.

***Caution!** These two shortcuts work only for triangles.*

31. Are the triangles in Figure 49 similar?

Example 31

You may find these Examples useful while doing the homework for this section.

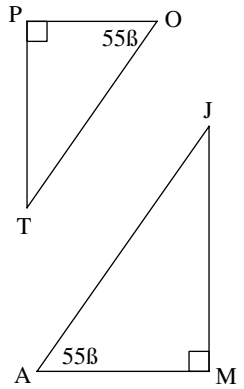


Figure 49

Each triangle has an angle with measure 55°. Each triangle has an angle with measure 90°.

So by Shortcut #1, the triangles are similar.

Example 32

32. Write a similarity statement for the triangles in Figure 49.

$\angle O$ corresponds to $\angle A$.
$\angle P$ corresponds to $\angle M$.
One similarity statement for these triangles is $\Delta TOP \sim \Delta JAM$.

Example 33

33. Are the triangles in Figure 50 similar?

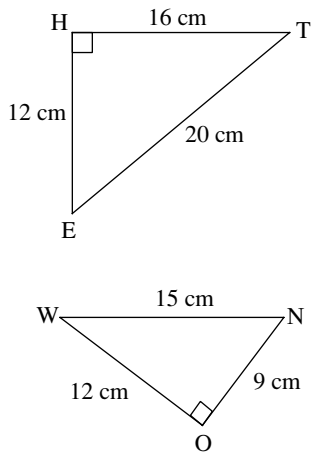


Figure 50

$$\frac{TH}{WO} = \frac{16}{12} = \frac{4}{3}$$

$$\frac{TE}{WN} = \frac{20}{15} = \frac{4}{3}$$

$$\frac{HE}{ON} = \frac{12}{9} = \frac{4}{3}$$

All corresponding lengths have the same ratio.

So, by Shortcut #2, the triangles are similar.

Example 34

34. Write a similarity statement for the triangles in Figure 50.

$\overline{TH}$ corresponds to $\overline{WO}$, so T corresponds to W and H corresponds to O.
$\overline{TE}$ corresponds to $\overline{WN}$, so E corresponds to N.
One similarity statement for these triangles is $\Delta THE \sim \Delta WON$.

Measures of Corresponding Angles of Similar Triangles

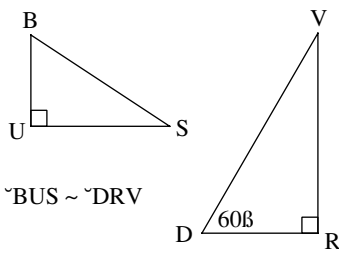
To find missing angle measures in similar triangles, you can apply what you have learned about triangles.

- The angle measures of a triangle add to 180˚.
- In similar triangles, corresponding angles have the same measure.
- In similar triangles, the order of the letters in a similarity statement matches the corresponding angles.

Example 35

You may find these Examples useful while doing the homework for this section.

35. The triangles in Figure 51 are similar. ΔBUS ~ ΔDRV. Find the missing angle measures.



˘BUS ~ ˘DRV

Figure 51

In ΔDRV, one way to find the measure of angle V is to use what you know about the sum of the angle measures in a triangle.

- *Add the given angle measures.* $60^\circ + 90^\circ = 150^\circ$
- *Subtract that result from 180˚.* $180^\circ - 150^\circ = 30^\circ$

So, m∠V = 30˚.

One way to find the measure of angle B in ΔBUS is to use the order of the letters in the similarity statement, ΔBUS ~ ΔDRV.

Angle B corresponds to angle D.
In similar triangles, corresponding angles have the same measure.
So, m∠B = m∠D = 60˚.
In the same way, angle S has the same measure as angle V.
So, m∠S = m∠V =30˚.

Lengths of Corresponding Sides of Similar Triangles

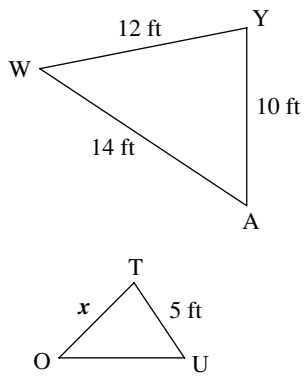
To find missing lengths in similar triangles, you can apply what you have learned about similar triangles.

- In similar triangles, lengths of corresponding sides have the same ratio.
- In similar triangles, the order of the letters in a similarity statement matches the corresponding sides.

You may find these Examples useful while doing the homework for this section.

Example 36

36. The triangles in Figure 52 are similar. ΔWAY ~ ΔOUT. Find x, the length of $\overline{OT}$.



˘WAY ~ ˘OUT

Figure 52

Here's one way to find x.

- *Corresponding lengths have the same ratio.* $\frac{AY}{UT} = \frac{WY}{OT}$
- *We know AY, UT, and WY.* $\frac{10}{5} = \frac{12}{x}$
- *Cross multiply.* $10 \cdot x = 5 \cdot 12$, $10x = 60$
- *To get x by itself, divide both sides by 10.* $\frac{10x}{10} = \frac{60}{10}$, $x = 6$

*So, **x**, the length of $\overline{OT}$ is 6 ft.*

AY is the length of line segment $\overline{AY}$. UT is the length of line segment $\overline{UT}$, and so on.

Example 37

37. In Figure 53, triangle ABC and triangle ADE are similar. Find ***y***, the length of $\overline{DE}$.

Notice that the triangles in this figure "overlap." Look carefully at the similarity statement to find pairs of corresponding angles and corresponding lengths.

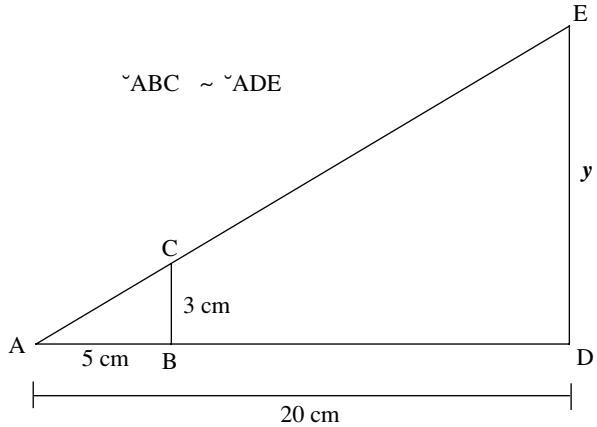


Figure 53

Here's one way to find ***y****.*

- *Corresponding lengths have the same ratio.* $\frac{AB}{AD} = \frac{BC}{DE}$
- *We know AB, AD, and BC.* $\frac{5}{20} = \frac{3}{y}$
- *Cross multiply.* $5 \cdot y = 3 \cdot 20$

  $5y = 60$
- *To get y by itself, divide both sides by 5.* $\frac{5y}{5} = \frac{60}{5}$

  $y = 12$

So, ***y****, the length of* $\overline{DE}$ *is 12 cm.*

Example 38

38. In Figure 54, triangle CRN is similar to triangle COB. Find ***x***, the length of $\overline{NB}$.

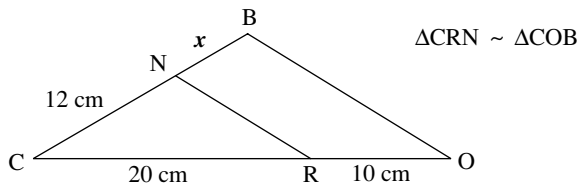


Figure 54

Here's one way to find ***x****.*

Notice that here, ***x*** *is the length of only a part of one side of* ΔCOB.

- *Corresponding lengths have the same ratio.* $\frac{CR}{CO} = \frac{CN}{CB}$
- *We know CR and CN.* $CO = 20 + 10 = 30$, $CB = x + 12$ $\frac{20}{30} = \frac{12}{x + 12}$
- *Cross multiply.* $20(x + 12) = 30 \cdot 12$

  $20(x + 12) = 360$
- *To remove the parentheses, use the distributive property.* $20x + 240 = 360$
- *Subtract 240 from both sides.* $20x + 240 - 240 = 360 - 240$

  $20x = 120$
- *To get x by itself, divide both sides by 20.* $\frac{20x}{20} = \frac{120}{20}$

  $x = 6$

So, ***x****, the length of* $\overline{NB}$ *is 6 cm.*

Explore

This Explore contains two investigations.

- **Congruent Triangles**
- **"Door to Door"**

You have been introduced to these investigations in the Explore module of this lesson on the computer. You can complete them using the information given here.

Investigation 1: Congruent Triangles

For this investigation, you will need a ruler and a protractor.

You have learned that two triangles are congruent if all corresponding sides have the same length and all corresponding angles have the same measure.

Often you don't need to verify all of the above information to know that two triangles are congruent. In this investigation you will test six ways for using less information to prove that two triangles are congruent. Some of these ways are true and some are false.

1. True or False: **Two triangles are congruent if they have one pair of congruent sides.**

   a. Draw a triangle and label its vertices A, B, and C.

   b. Measure the angles of ΔABC with a protractor. Measure the sides with a ruler. Record your measurements on the triangle.

   c. See if you can draw another triangle (ΔDEF) with both of these features:

      • One side of ΔDEF is congruent to one side of ΔABC, and
      • ΔDEF is **not** congruent to ΔABC.

   d. Based on your investigation in (c), do you think the conjecture is true? Discuss what you tried in (c) and how you decided whether the conjecture is true.

2. True or False: **Two triangles are congruent if they have two pairs of congruent sides.**

   a. Draw a triangle and label its vertices A, B, and C.

   b. Measure the angles of ΔABC with a protractor. Measure the sides with a ruler. Record your measurements on the triangle.

   c. See if you can draw another triangle (ΔDEF) with both of these features:

      • Two sides of ΔDEF are congruent to two sides of ΔABC, and
      • ΔDEF is **not** congruent to ΔABC.

   d. Based on your investigation in (c), do you think the conjecture is true? Discuss what you tried in (c) and how you decided whether the conjecture is true.

3. True or False: **Two triangles are congruent if they have three pairs of congruent sides.**

   a. Draw a triangle and label its vertices A, B, and C.

   b. Measure the angles of ΔABC with a protractor. Measure the sides with a ruler. Record your measurements on the triangle.

c. See if you can draw another triangle (ΔDEF) with both of these features:

- Three sides of ΔDEF are congruent to three sides of ΔABC, and
- ΔDEF is **not** congruent to ΔABC.

d. Based on your investigation in (c), do you think the conjecture is true? Discuss what you tried in (c) and how you decided whether the conjecture is true.

4. True or False: **Two triangles are congruent if they have three pairs of congruent angles.**

a. Draw a triangle and label its vertices A, B, and C.

b. Measure the angles of ΔABC with a protractor. Measure the sides with a ruler. Record your measurements on the triangle.

c. See if you can draw another triangle (ΔDEF) with both of these features:

- Three angles of ΔDEF are congruent to three angles of ΔABC, and
- ΔDEF is **not** congruent to ΔABC.

d. Based on your investigation in (c), do you think the conjecture is true? Discuss what you tried in (c) and how you decided whether the conjecture is true.

5. True or False: **Two triangles are congruent if they have two pairs of congruent sides and the angles formed by those sides are also congruent.**

   a. Draw a triangle and label its vertices A, B, and C.

   b. Measure the angles of ΔABC with a protractor. Measure the sides with a ruler. Record your measurements on the triangle.

   c. See if you can draw another triangle (ΔDEF) with both of these features:

      • Two sides of ΔDEF are congruent to two sides of ΔABC, and
      • The angles formed by those sides are congruent.
      • ΔDEF is **not** congruent to ΔABC.

   d. Based on your investigation in (c), do you think the conjecture is true? Discuss what you tried in (c) and how you decided whether the conjecture is true.

6. True and False: **Two triangles are congruent if they have two pairs of congruent angles and the sides shared by those angles are also congruent.**

   a. Draw a triangle and label its vertices A, B, and C.

   b. Measure the angles of ΔABC with a protractor. Measure the sides with a ruler. Record your measurements on the triangle.

c. See if you can draw another triangle (ΔDEF) with both of these features:

- Two angles of ΔDEF are congruent to two angles of ΔABC, and
- The sides shared by those angles are congruent.
- ΔDEF is **not** congruent to ΔABC.

d. Based on your investigation in (c), do you think the conjecture is true? Discuss what you tried in (c) and how you decided whether the conjecture is true.

Investigation 2: "Door to Door"

1. First, sketch a copy of the rectangle in Figure 55. Then sketch a larger rectangle which is similar to the first rectangle and has the property that corresponding lengths of sides are in the ratio $\frac{2}{1}$.

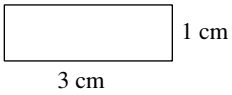


Figure 55

a. Find the perimeter of each rectangle.

b. What is the ratio of the perimeters? (Put the perimeter of the larger rectangle in the numerator.) How does the ratio of the perimeters compare to the ratio of the corresponding lengths?

c. Sketch another larger rectangle similar to the rectangle in Figure 55, so that the ratio of corresponding lengths is $\frac{3}{1}$.

d. Find the perimeter of each rectangle.

e. What is the ratio of the perimeters? (Put the perimeter of the larger rectangle in the numerator.) How does the ratio of the perimeters compare to the ratio of the corresponding lengths?

f. Look back at your results in (b) and (e). Do you see a pattern? Test your ideas by sketching some more similar shapes and finding the ratio of their perimeters.

2. Sketch a larger rectangle similar to the rectangle in Figure 56, so that the ratio of their corresponding lengths is $\frac{2}{1}$.

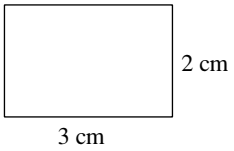


Figure 56

a. Find the area of each rectangle.

b. What is the ratio of the areas? (Put the area of the larger rectangle in the numerator.) How does the ratio of the areas compare to the ratio of the corresponding lengths?

c. Sketch another larger rectangle similar to the rectangle in Figure 56, so that the ratio of the corresponding lengths is $\frac{3}{1}$.

d. Find the area of each rectangle.

e. What is the ratio of the areas? (Put the area of the larger rectangle in the numerator.) How does the ratio of the areas compare to the ratio of the corresponding lengths?

f. Look back at your results in (b) and (e). Do you see a pattern? Test your ideas by sketching some more similar shapes and finding the ratio of their areas.

3. Sketch a rectangular prism similar to the one in Figure 57, so that the ratio of their corresponding lengths is $\frac{2}{1}$.

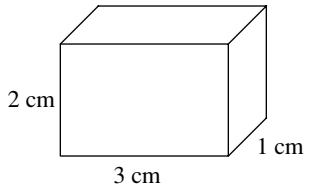


Figure 57

a. Find the volume of each rectangular prism.

b. What is the ratio of the volumes? (Put the volume of the larger prism in the numerator.) How does the ratio of the volumes compare to the ratio of the corresponding lengths?

c. Sketch another rectangular prism similar to the one in Figure 57, so that the ratio of corresponding lengths is $\frac{3}{1}$.

d. Find the volume of each prism.

e. What is the ratio of the volumes? (Put the volume of the larger prism in the numerator.) How does the ratio of the volumes compare to the ratio of the corresponding lengths?

f. Look back at your results in (b) and (e). Do you see a pattern? Test your ideas by sketching some more similar prisms and finding the ratio of their volumes.

Homework

Concept 1: Triangles and Parallelograms

The Sum of the Angle Measures of a Triangle

For help working these types of problems, go back to Examples 1–3 in the Explain section of this lesson.

1. In Figure 58, find the measure of ∠W.

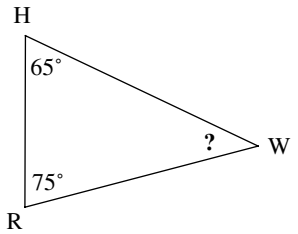


Figure 58

2. In Figure 59, find the measure of ∠T.

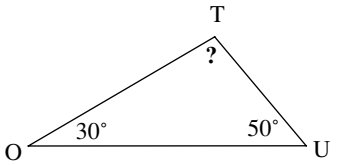


Figure 59

3. In Figure 60, find the measure of ∠N.

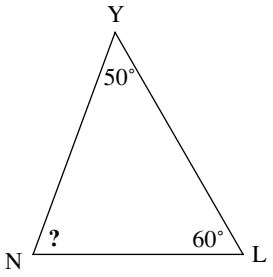


Figure 60

4. In Figure 61, find the measure of ∠E.

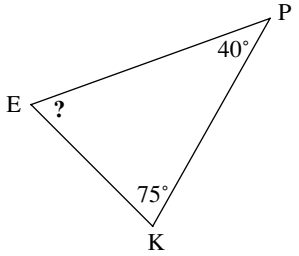


Figure 61

5. In Figure 62, find the measure of ∠Q.

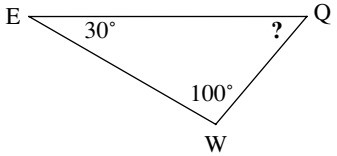


Figure 62

6. In Figure 63, find the measure of ∠M.

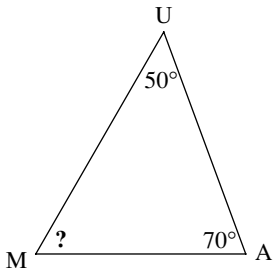


Figure 63

7. In Figure 64, find the measure of ∠I.

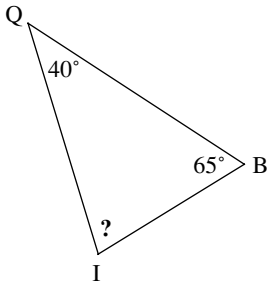


Figure 64

8. In Figure 65, find the measure of ∠S.

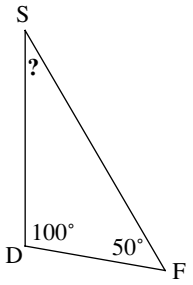


Figure 65

9. In Figure 66, find the measure of ∠A.

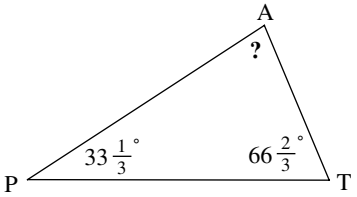


Figure 66

10. In Figure 67, find the measure of ∠R.

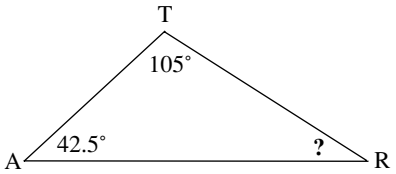


Figure 67

11. In Figure 68, find the measure of ∠D.

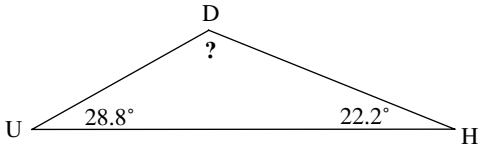


Figure 68

12. In Figure 69, find the measure of ∠T.

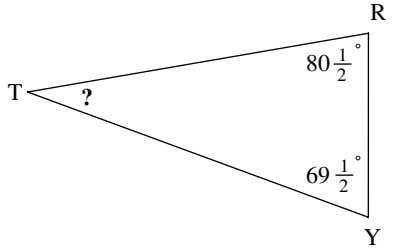


Figure 69

13. In Figure 70, find the measure of ∠Z.

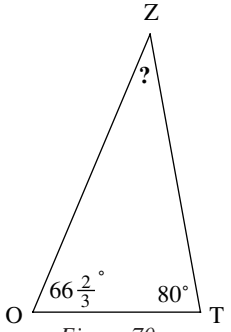


Figure 70

14. In Figure 71, find the measure of ∠W.

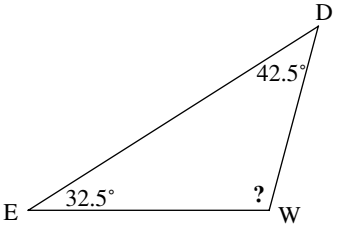


Figure 71

15. In Figure 72, find the measure of ∠B.

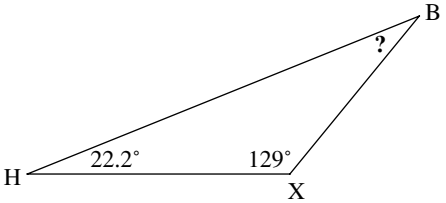


Figure 72

16. In Figure 73, find the measure of ∠T.

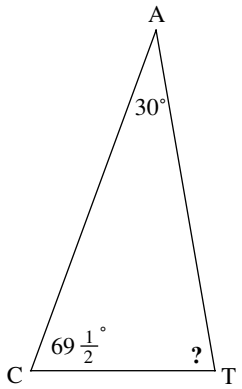


Figure 73

17. In Figure 74, find the measure of ∠BAT.

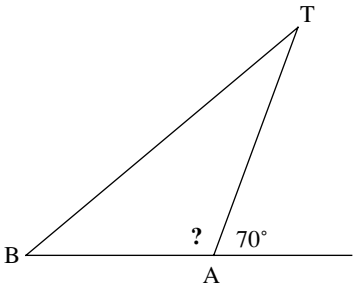


Figure 74

18. In Figure 75, find the measure of ∠AGS.

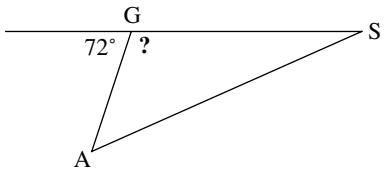


Figure 75

19. In Figure 76, find the measure of ∠GLO.

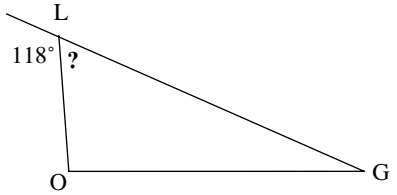


Figure 76

20. In Figure 77, find the measure of ∠EVM.

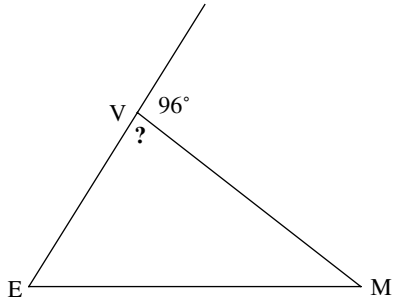


Figure 77

21. In Figure 78, find the measure of ∠N.

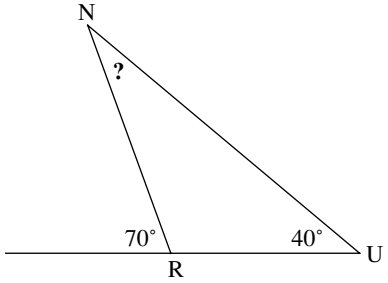


Figure 78

22. In Figure 79, find the measure of ∠M.

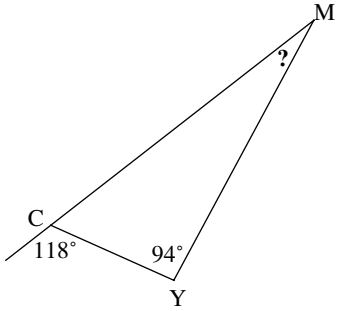


Figure 79

23. In Figure 80, find the measure of ∠Y.

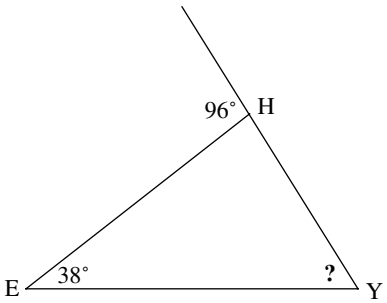


Figure 80

24. In Figure 81, find the measure of ∠E.

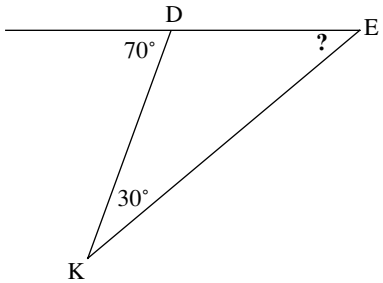


Figure 81

Congruent Triangles

For help working these types of problems, go back to Examples 4–8 in the Explain section of this lesson.

25. In Figure 82, ΔABC is congruent to ΔDEF. Which angle of ΔDEF is congruent to ∠B?

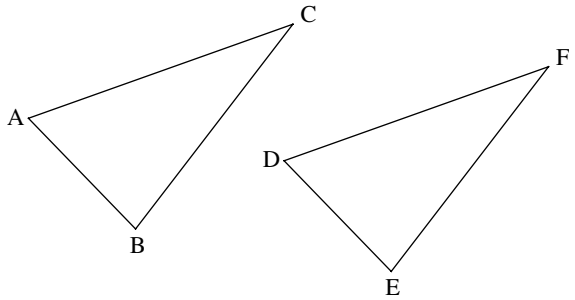


Figure 82

26. In Figure 83, ΔBCD is congruent to ΔEFG. Which angle of ΔEFG is congruent to ∠D?

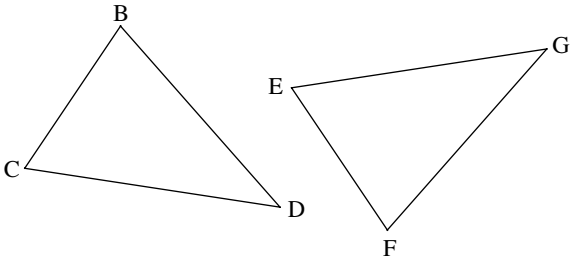


Figure 83

27. In Figure 84, ΔCDE is congruent to ΔFGH. Which angle of ΔCDE is congruent to ∠F?

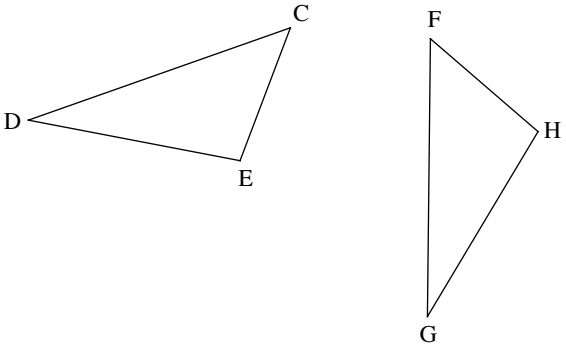


Figure 84

28. In Figure 85, ΔDEF is congruent to ΔGHI. Which angle of ΔDEF is congruent to ∠I?

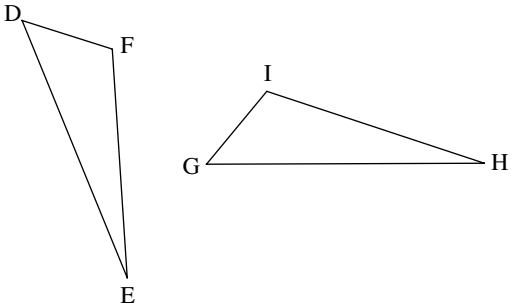


Figure 85

29. In Figure 86, ΔABC is congruent to ΔDEF. Which side of ΔDEF is congruent to $\overline{AB}$?

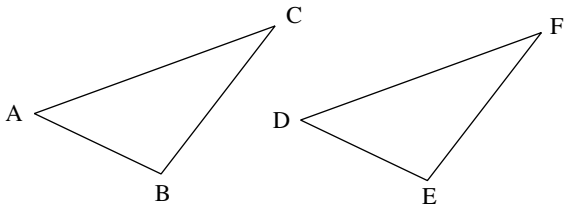


Figure 86

30. In Figure 87, ΔBCD is congruent to ΔEFG. Which side of ΔBCD is congruent to $\overline{EG}$?

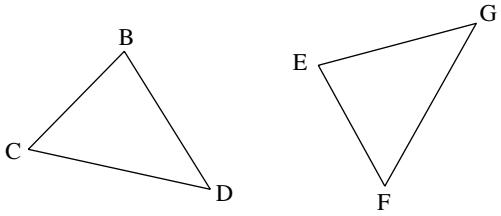


Figure 87

31. In Figure 88, ΔCDE is congruent to ΔFGH. Which side of ΔCDE is congruent to $\overline{GH}$?

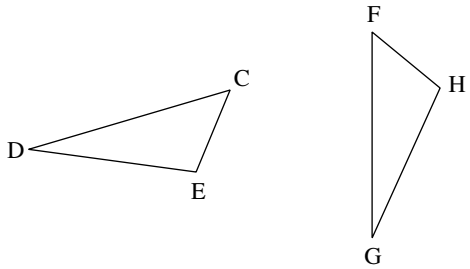


Figure 88

32. In Figure 89, ΔDEF is congruent to ΔGHI.
Which side of ΔGHI is congruent to $\overline{DF}$?

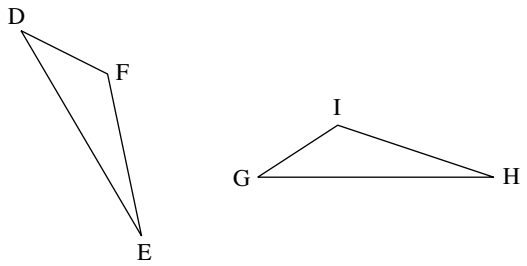


Figure 89

33. In Figure 90, the triangles are congruent.
Write a congruence relation for these triangles.

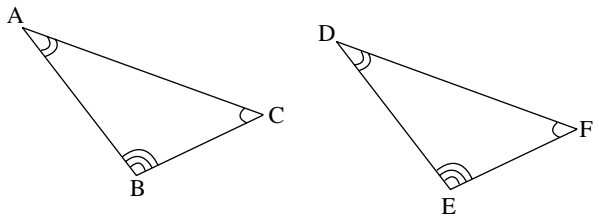


Figure 90

34. In Figure 91, the triangles are congruent.
Write a congruence relation for these triangles.

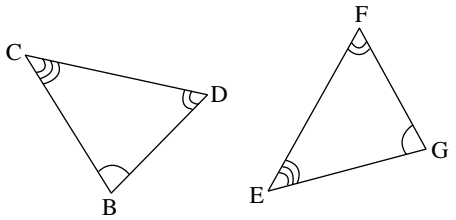


Figure 91

35. In Figure 92, the triangles are congruent.
Write a congruence relation for these triangles.

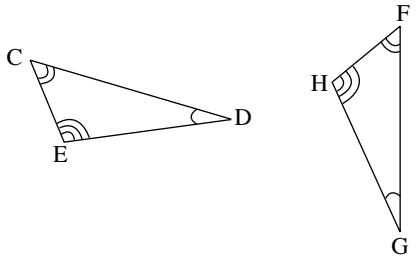


Figure 92

36. In Figure 93, the triangles are congruent.
Write a congruence relation for these triangles.

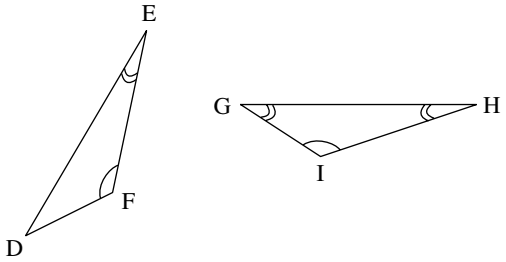


Figure 93

37. In Figure 94, ΔCDE is congruent to ΔFGH.
Find the measure of ∠G.

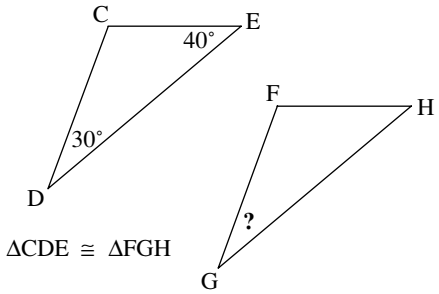


Figure 94

38. In Figure 95, ΔDEF is congruent to ΔGHI.
Find the measure of ∠H.

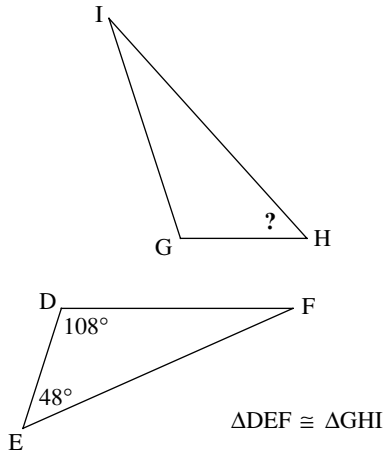


Figure 95

39. In Figure 96, ΔEFG is congruent to ΔHIJ. Find the measure of ∠I.

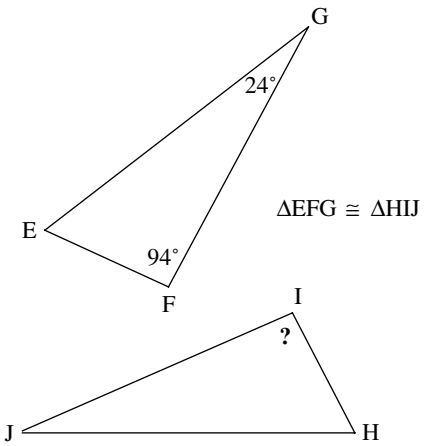


Figure 96

40. In Figure 97, ΔFGH is congruent to ΔIJK. Find the measure of ∠H.

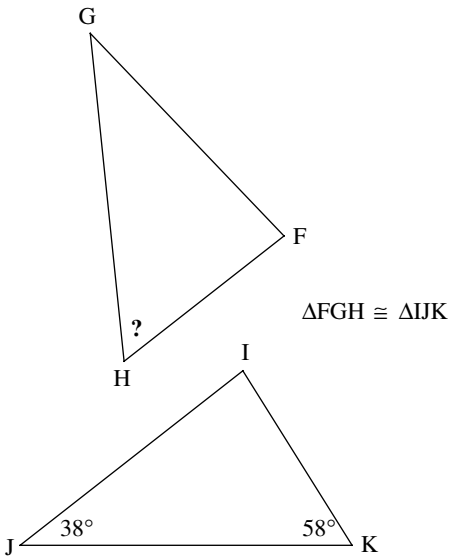


Figure 97

41. In Figure 98, ΔCDE is congruent to ΔFGH. Find the measure of ∠F.

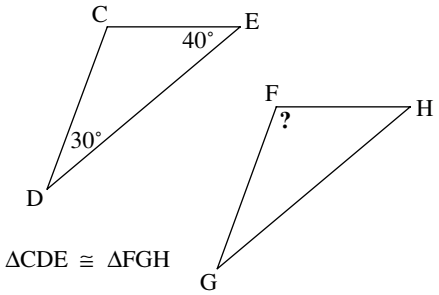


Figure 98

42. In Figure 99, ΔDEF is congruent to ΔGHI. Find the measure of ∠I.

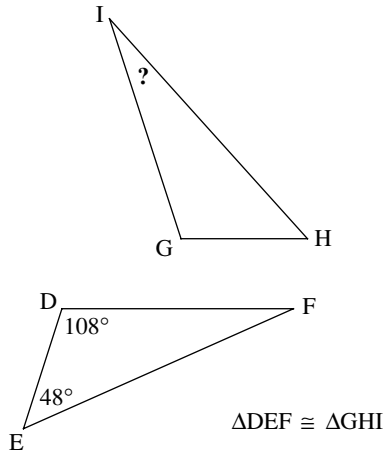


Figure 99

43. In Figure 100, ΔEFG is congruent to ΔHIJ. Find the measure of ∠H.

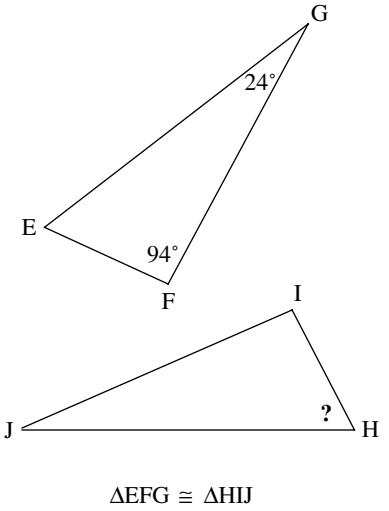


Figure 100

44. In Figure 101, ΔFGH is congruent to ΔIJK. Find the measure of ∠F.

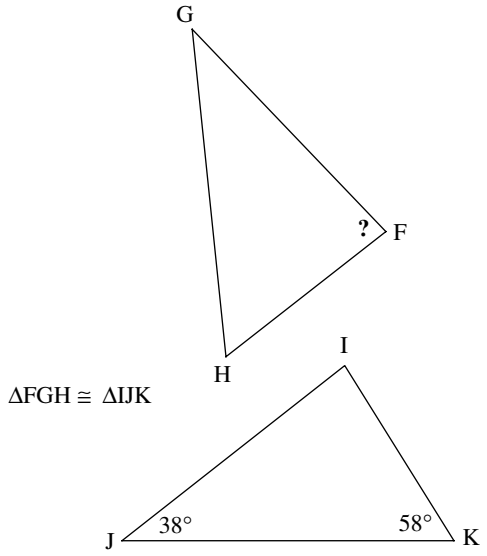


Figure 101

45. In Figure 102, ΔSRK is congruent to ΔFIN. Find the length of $\overline{SR}$.

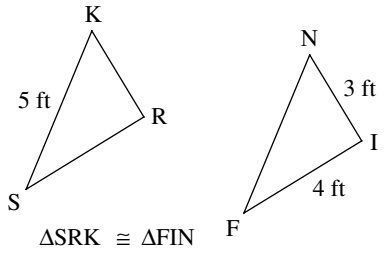


Figure 102

46. In Figure 103, ΔCDE is congruent to ΔFGH. Find the length of $\overline{FG}$.

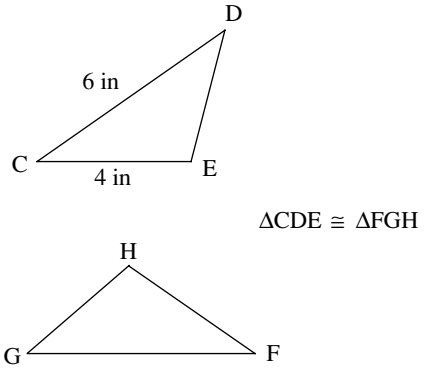


Figure 103

47. In Figure 104, ΔKLM is congruent to ΔNOP. Find the length of $\overline{LM}$.

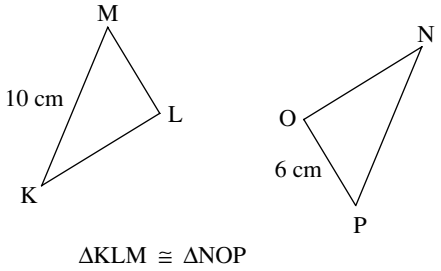


Figure 104

48. In Figure 105, ΔSTU is congruent to ΔVWX. Find the length of $\overline{SU}$.

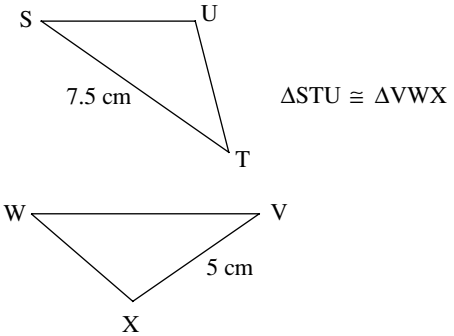


Figure 105

Isosceles Triangles and Equilateral Triangles

For help working these types of problems, go back to Examples 9–14 in the Explain section of this lesson.

49. In Figure 106, ΔRIT is an isosceles triangle. Which pair of angles **must** have the same measure?

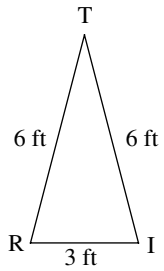


Figure 106

50. In Figure 107, ΔBSU is an isosceles triangle. Which pair of angles **must** have the same measure?

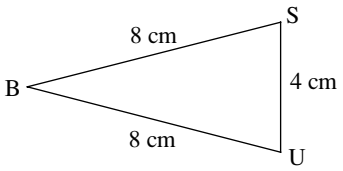


Figure 107

51. In Figure 108, ΔWAY is an isosceles triangle. Which pair of angles **must** have the same measure?

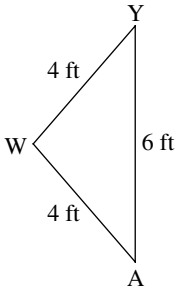


Figure 108

52. In Figure 109, ΔTOY is an isosceles triangle. Which pair of angles **must** have the same measure?

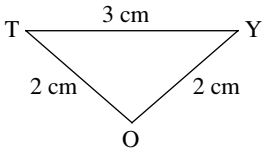


Figure 109

53. In Figure 110, ΔBAT is an isosceles triangle. Find the measure of ∠T.

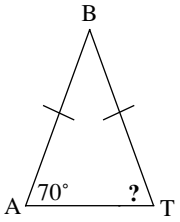


Figure 110

54. In Figure 111, ΔJET is an isosceles triangle. Find the measure of ∠E.

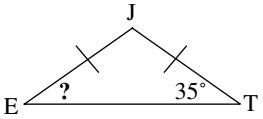


Figure 111

55. In Figure 112, ΔWHO is an isosceles triangle. Find the measure of ∠W.

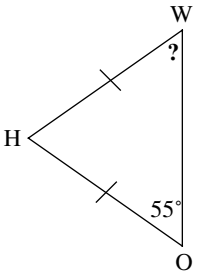


Figure 112

56. In Figure 113, ΔSAM is an isosceles triangle. Find the measure of ∠S.

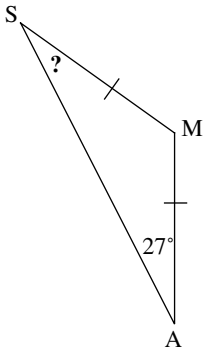


Figure 113

57. In Figure 114, find the length of $\overline{BA}$.

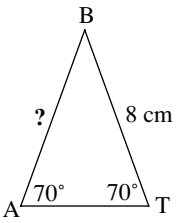


Figure 114

58. In Figure 115, find the length of $\overline{JT}$.

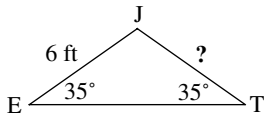


Figure 115

59. In Figure 116, find the length of $\overline{HO}$.

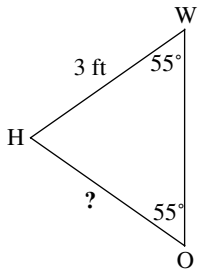


Figure 116

60. In Figure 117, find the length of $\overline{AM}$.

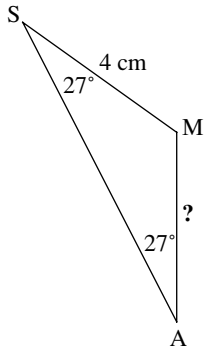


Figure 117

61. Sketch an isosceles triangle, ΔPUT, with $\overline{PU}$ and $\overline{PT}$ of equal lengths.

62. Sketch an isosceles triangle, ΔCAN, with $\overline{CA}$ and $\overline{AN}$ of different lengths.

63. Sketch an isosceles triangle, ΔRAT, with all sides of equal lengths.

64. Sketch an isosceles triangle, ΔSLO, with $\overline{LO}$ and $\overline{SO}$ of equal lengths.

65. In Figure 122, ΔASK is an equilateral triangle. Find the length of $\overline{AS}$.

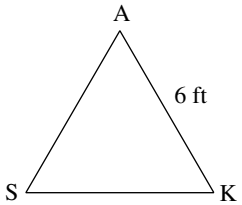


Figure 122

66. In Figure 123, ΔPAL is an equilateral triangle. Find the length of $\overline{AL}$.

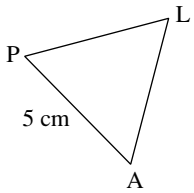


Figure 123

67. In Figure 124, ΔJAM is an equilateral triangle. Find the length of $\overline{AM}$.

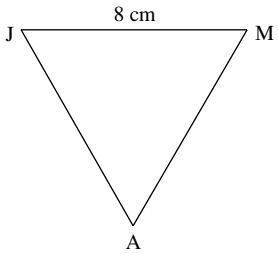


Figure 124

68. In Figure 125, ΔBDU is an equilateral triangle. Find the length of $\overline{BU}$.

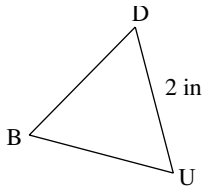


Figure 125

69. In Figure 126, find the measure of angle S.

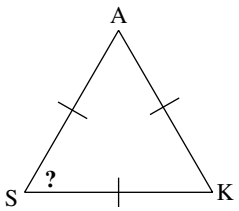


Figure 126

70. In Figure 127, find the measure of angle A.

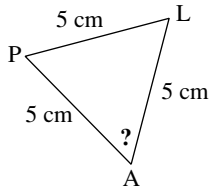


Figure 127

71. In Figure 128, find the measure of angle M.

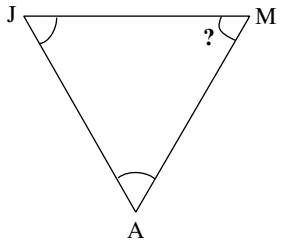


Figure 128

72. In Figure 129, find the measure of angle U.

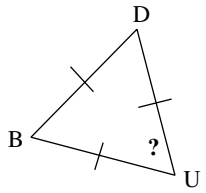


Figure 129

Right Triangles and the Pythagorean Theorem

For help working these types of problems, go back to Examples 15–19 in the Explain section of this lesson.

73. In a right triangle, the measure of one angle is 28°. Find the measures of the other angles.

74. In a right triangle, the measure of one angle is 80°. Find the measures of the other angles.

75. In a right triangle, the measure of one angle is 30°. Find the measures of the other angles.

76. In a right triangle, the measure of one angle is 45°. Find the measures of the other angles.

77. In a certain right triangle, the lengths of the sides are 60 cm, 63 cm, and 87 cm. Find the length of the hypotenuse.

78. In a certain right triangle, the lengths of the sides are 15 ft, 20 ft, and 25 ft. Find the length of the hypotenuse.

79. In a certain right triangle, the lengths of the sides are 25 in, 60 in, and 65 in. Find the lengths of the legs.

80. In a certain right triangle, the lengths of the sides are 5m, 12m, and 13m. Find the lengths of the legs.

81. In ΔSTW, the lengths of the sides are 3 cm, 4 cm, and 5 cm. Is ΔSTW a right triangle?

82. In ΔLUG, the lengths of the sides are 5 ft, 11 ft, and 13 ft. Is ΔLUG a right triangle?

83. In ΔPTA, the lengths of the sides are 9 cm, 10 cm, and 12 cm. Is ΔPTA a right triangle?

84. In ΔTIP, the lengths of the sides are 10 ft, 24 ft, and 26 ft. Is ΔTIP a right triangle?

85. In ΔBIN, the lengths of the sides are 10 cm, 12 cm, and 24 cm. Is ΔBIN a right triangle?

86. In ΔCOW, the lengths of the sides are 15 in, 36 in, and 39 in. Is ΔCOW a right triangle?

87. In ΔIRT, the lengths of the sides are 60 cm, 63 cm, and 87 cm. Is ΔIRT a right triangle?

88. In ΔMTA, the lengths of the sides in are 5 ft, 7 ft, and 9 ft. Is ΔMTA a right triangle?

89. The triangle in Figure 130 is a right triangle. Find *c*, the length of its hypotenuse.

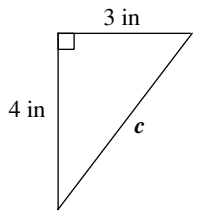


Figure 130

90. The triangle in Figure 131 is a right triangle. Find *c*, the length of its hypotenuse.

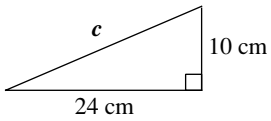


Figure 131

91. The triangle in Figure 132 is a right triangle. Find c, the length of its hypotenuse.

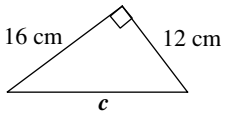


Figure 132

92. The triangle in Figure 133 is a right triangle. Find c, the length of its hypotenuse.

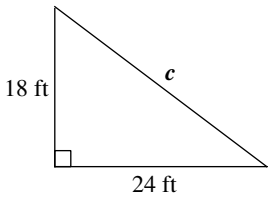


Figure 133

93. The triangle in Figure 134 is a right triangle. Find b, the length of one of its legs.

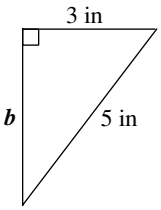


Figure 134

94. The triangle in Figure 135 is a right triangle. Find a, the length of one of its legs.

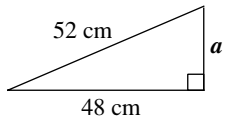


Figure 135

95. The triangle in Figure 136 is a right triangle. Find a, the length of one of its legs.

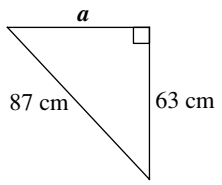


Figure 136

96. The triangle in Figure 137 is a right triangle. Find b, the length of one of its legs.

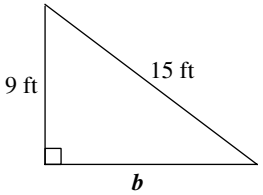


Figure 137

Parallel Lines and Parallelograms

For help working these types of problems, go back to Examples 20–23 in the Explain section of this lesson.

97. In Figure 138, $k \parallel r$. The measure of angle 1 is 75°. Find the measure of angle 2.

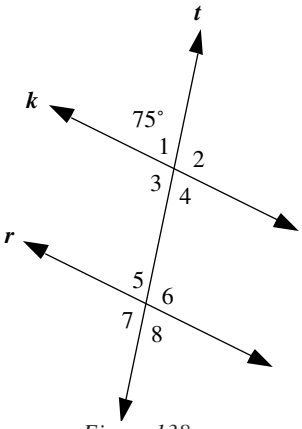


Figure 138

98. In Figure 138, $k \parallel r$. The measure of angle 1 is 75°. Find the measure of angle 3.

99. In Figure 138, $k \parallel r$. The measure of angle 1 is 75°. Find the measure of angle 4.

100. In Figure 138, $k \parallel r$. The measure of angle 1 is 75°. Find the measure of angle 5.

101. In Figure 138, $k \parallel r$. The measure of angle 1 is 75°. Find the measure of angle 6.

102. In Figure 138, $k \parallel r$. The measure of angle 1 is 75°. Find the measure of angle 7.

103. In Figure 138, $k \parallel r$. The measure of angle 1 is 75°. Find the measure of angle 8.

104. In Figure 138, $k \parallel r$. Are angles 1 and 5 supplementary angles, corresponding angles, or vertical angles?

105. In Figure 139, line segment $\overline{AB}$ is parallel to line segment $\overline{ED}$. Find the measure of angle A.

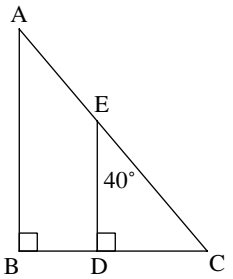


Figure 139

106. In Figure 139, line segment $\overline{AB}$ is parallel to line segment $\overline{ED}$. Find the measure of angle C.

107. In Figure 140, line segment $\overline{TU}$ is parallel to line segment $\overline{VW}$. Find the measure of angle STU.

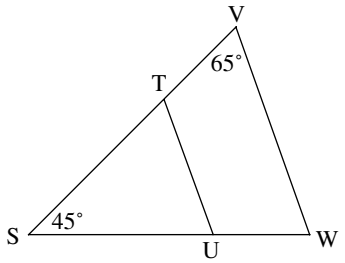


Figure 140

108. In Figure 140, line segment $\overline{TU}$ is parallel to line segment $\overline{VW}$. Find the measure of angle SUT.

109. In Figure 140, line segment $\overline{TU}$ is parallel to line segment $\overline{VW}$. Find the measure of angle W.

110. In Figure 141, line segment $\overline{KL}$ is parallel to line segment $\overline{MN}$. Find the measure of angle M.

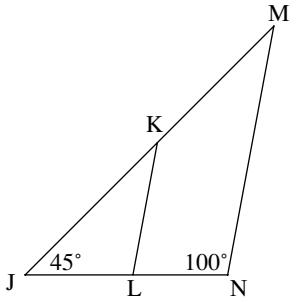


Figure 141

111. In Figure 141, line segment $\overline{KL}$ is parallel to line segment $\overline{MN}$. Find the measure of angle JKL.

112. In Figure 141, line segment $\overline{KL}$ is parallel to line segment $\overline{MN}$. Find the measure of angle JLK.

113. In Figure 142, the quadrilateral is a parallelogram. Find the measure of angle Q.

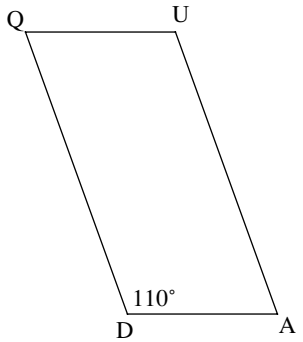


Figure 142

114. In Figure 142, the quadrilateral is a parallelogram. Find the measure of angle U.

115. In Figure 142, the quadrilateral is a parallelogram. Find the measure of angle A.

116. In Figure 143, the quadrilateral is a parallelogram. Find the measure of angle F.

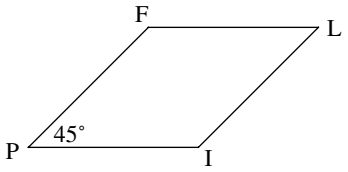


Figure 143

117. In Figure 144, the quadrilateral is a parallelogram. Find the length of $\overline{SH}$.

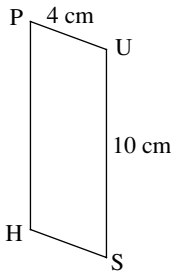


Figure 144

118. In Figure 144, the quadrilateral is a parallelogram. Find the length of $\overline{PH}$.

119. In Figure 145, the quadrilateral is a parallelogram. Find the length of $\overline{DE}$.

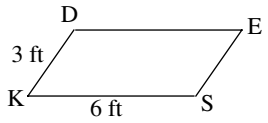


Figure 145

120. In Figure 145, the quadrilateral is a parallelogram. Find the length of $\overline{ES}$.

Concept 2: Similar Polygons

Recognizing Similar Polygons

For help working these types of problems, go back to Examples 24–27 in the Explain section of this lesson.

121. Are the polygons in Figure 146 similar polygons?

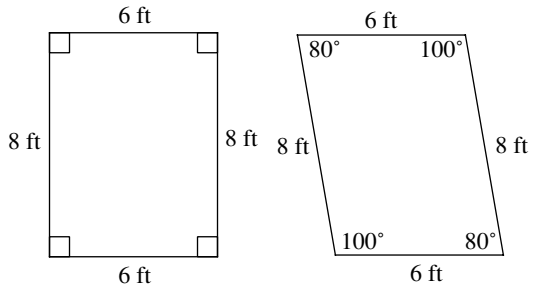


Figure 146

122. Are the rectangles in Figure 147 similar rectangles?

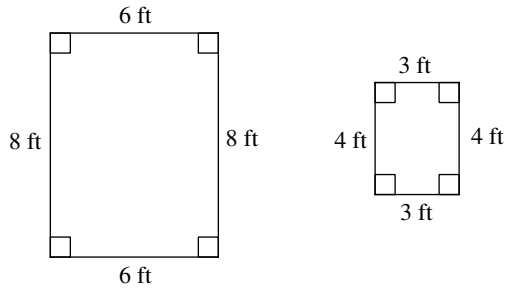


Figure 147

123. Are the polygons in Figure 148 similar polygons?

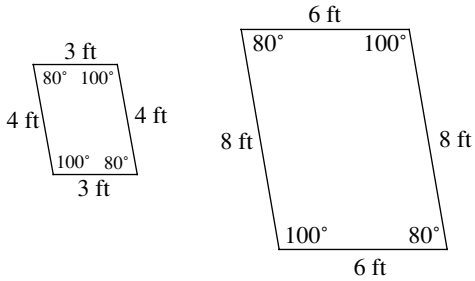


Figure 148

124. Are the polygons in Figure 149 similar polygons?

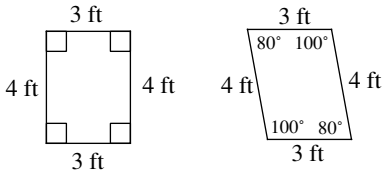


Figure 149

125. Are the rectangles in Figure 150 similar rectangles?

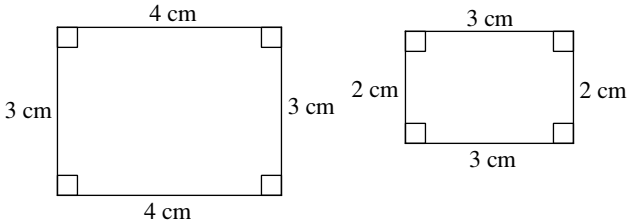


Figure 150

126. Are the triangles in Figure 151 similar triangles?

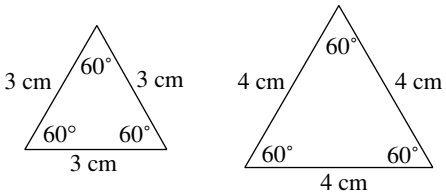


Figure 151

127. Are the hexagons in Figure 152 similar hexagons?

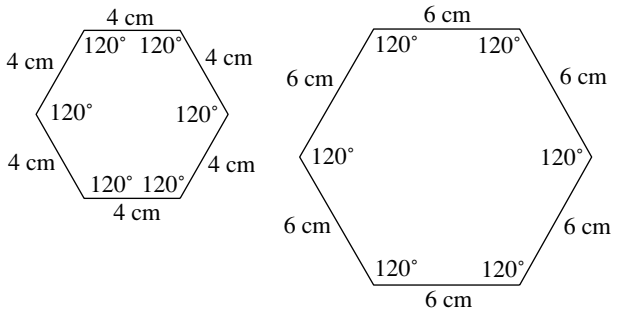


Figure 152

128. Are the pentagons in Figure 153 similar pentagons?

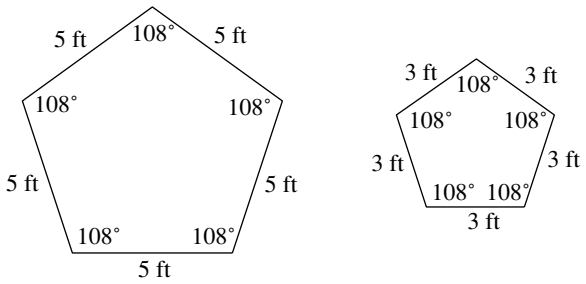
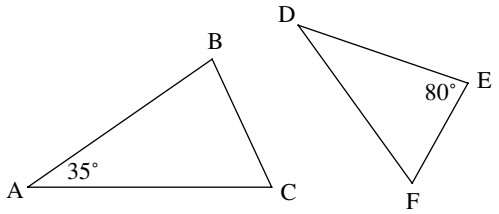


Figure 153

129. In Figure 154 , the triangles are similar. ΔABC ~ ΔDEF. Find the measure of angle D.



ΔABC ~ ΔDEF

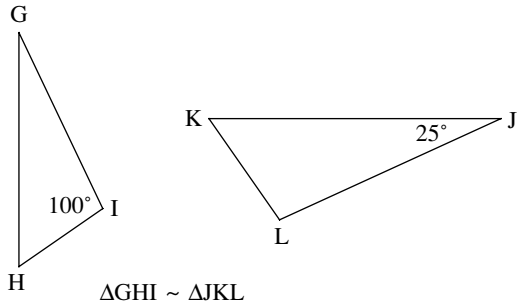
Figure 154

130. In Figure 154 , the triangles are similar. ΔABC ~ ΔDEF Find the measure of angle B.

131. In Figure 154 , the triangles are similar. ΔABC ~ ΔDEF. Find the measure of angle C.

132. In Figure 154 , the triangles are similar. ΔABC ~ ΔDEF. Find the measure of angle F.

133. In Figure 155, the triangles are similar. ΔGHI ~ ΔJKL. Find the measure of angle L.



ΔGHI ~ ΔJKL

Figure 155

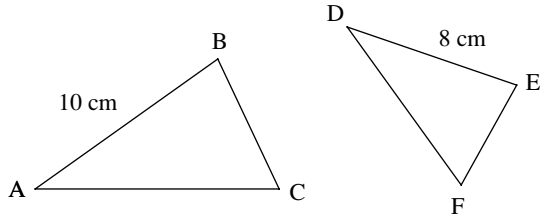
134. In Figure 155, the triangles are similar. ΔGHI ~ ΔJKL. Find the measure of angle G.

135. In Figure 155, the triangles are similar. ΔGHI ~ ΔJKL. Find the measure of angle H.

136. In Figure 155, the triangles are similar. ΔGHI ~ ΔJKL. Find the measure of angle K.

137. In Figure 156, the triangles are similar. ΔABC ~ ΔDEF. Find this ratio:

$$\frac{\text{length of } \overline{BC}}{\text{length of } \overline{EF}}$$



ΔABC ~ ΔDEF

Figure 156

138. In Figure 156, the triangles are similar. ΔABC ~ ΔDEF. Find this ratio:

$$\frac{\text{length of } \overline{DF}}{\text{length of } \overline{AC}}$$

139. In Figure 156, the triangles are similar. ΔABC ~ ΔDEF. Find this ratio:

$$\frac{\text{length of } \overline{EF}}{\text{length of } \overline{BC}}$$

140. In Figure 156, the triangles are similar. ΔABC ~ ΔDEF. Find this ratio:

$$\frac{\text{length of } \overline{AC}}{\text{length of } \overline{DF}}$$

141. In Figure 157, the triangles are similar. ΔGHI ~ ΔJKL. Find this ratio:

$$\frac{\text{length of } \overline{HI}}{\text{length of } \overline{KL}}$$

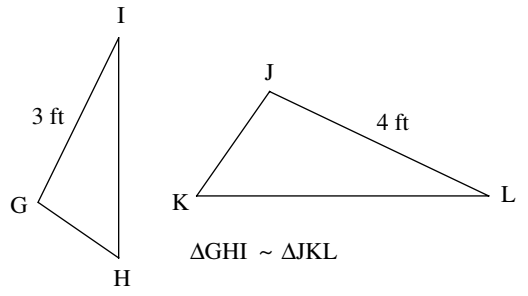


Figure 157

142. In Figure 157, the triangles are similar. ΔGHI ~ ΔJKL. Find this ratio:

$$\frac{\text{length of } \overline{JK}}{\text{length of } \overline{GH}}$$

143. In Figure 157, the triangles are similar. ΔGHI ~ ΔJKL. Find this ratio:

$$\frac{\text{length of } \overline{KL}}{\text{length of } \overline{HI}}$$

144. In Figure 157, the triangles are similar. ΔGHI ~ ΔJKL. Find this ratio:

$$\frac{\text{length of } \overline{GH}}{\text{length of } \overline{JK}}$$

Writing a Similarity Statement

For help working these types of problems, go back to Examples 28–30 in the Explain section of this lesson.

145. In Figure 158, the triangles are similar. Complete this similarity statement: ΔAFK ~ Δ_____

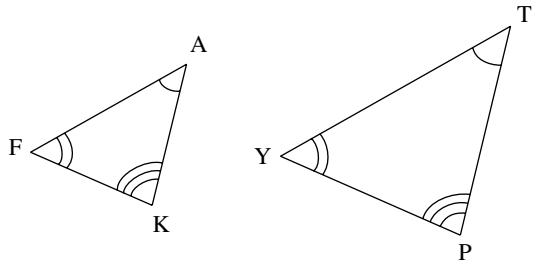


Figure 158

146. In Figure 158, the triangles are similar. Complete this similarity statement: ΔFAK ~ Δ_____

147. In Figure 158, the triangles are similar. Complete this similarity statement: ΔKAF ~ Δ_____

148. In Figure 158, the triangles are similar. Complete this similarity statement: ΔKFA ~ Δ_____

149. In Figure 159, the triangles are similar. Complete this similarity statement: ΔUSA ~ Δ_____

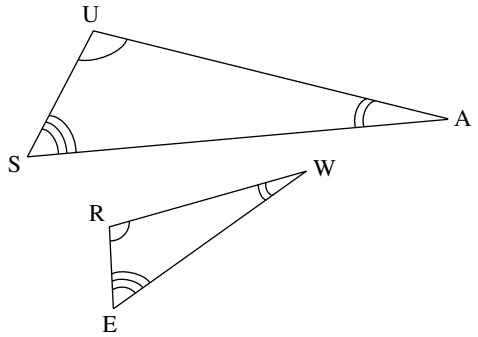


Figure 159

150. In Figure 159, the triangles are similar. Complete this similarity statement: ΔASU ~ Δ_____

151. In Figure 159, the triangles are similar. Complete this similarity statement: ΔSUA ~ Δ_____

152. In Figure 159, the triangles are similar. Complete this similarity statement: ΔUAS ~ Δ_____

153. ΔLAW ~ ΔYER
Find the angle in ΔLAW that has the same measure as angle Y.

154. ΔLAW ~ ΔYER
Find the angle in ΔLAW that has the same measure as angle E.

155. ΔLAW ~ ΔYER
Find the angle in ΔLAW that has the same measure as angle R.

156. ΔLAW ~ ΔYER
Find the angle in ΔYER that has the same measure as angle A.

157. ΔPUT ~ ΔDWN
Find the angle in ΔDWN that has the same measure as angle P.

158. ΔPUT ~ ΔDWN
Find the angle in ΔDWN that has the same measure as angle T.

159. ΔPUT ~ ΔDWN
Find the angle in ΔDWN that has the same measure as angle U.

160. ΔPUT ~ ΔDWN
Find the angle in ΔPUT that has the same measure as angle D.

161. ΔLAW ~ ΔYER
The ratio of LA to YE is 7 to 3. Find the ratio of AW to ER.

162. ΔLAW ~ ΔYER
The ratio of LA to YE is 7 to 3. Find the ratio of LW to YR.

163. ΔLAW ~ ΔYER
The ratio of LA to YE is 7 to 3. Find the ratio of ER to AW.

164. ΔLAW ~ ΔYER
The ratio of LA to YE is 7 to 3. Find the ratio of YR to LW.

165. ΔCAT ~ ΔNIP
The ratio of CT to NP is 9 to 4. Find the ratio of CA to NI.

166. ΔCAT ~ ΔNIP
The ratio of CT to NP is 9 to 4. Find the ratio of AT to IP.

167. ΔCAT ~ ΔNIP
The ratio of CT to NP is 9 to 4. Find the ratio of NI to CA.

168. ΔCAT ~ ΔNIP
The ratio of CT to NP is 9 to 4. Find the ratio of IP to AT.

Shortcuts for Recognizing Similar Triangles

For help working these types of problems, go back to Examples 31–34 in the Explain section of this lesson.

169. Are the triangles in Figure 160 similar?

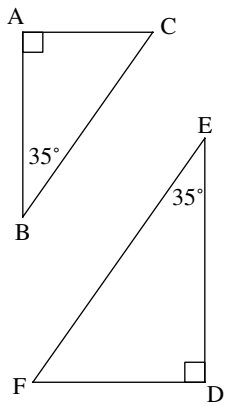


Figure 160

170. Are the triangles in Figure 161 similar?

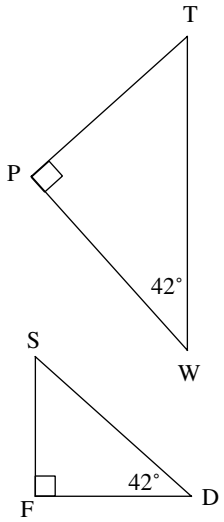


Figure 161

171. Are the triangles in Figure 162 similar?

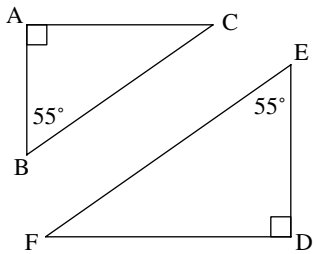


Figure 162

172. Are the triangles in Figure 163 similar?

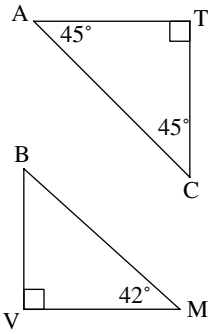


Figure 163

173. Are the triangles in Figure 164 similar?

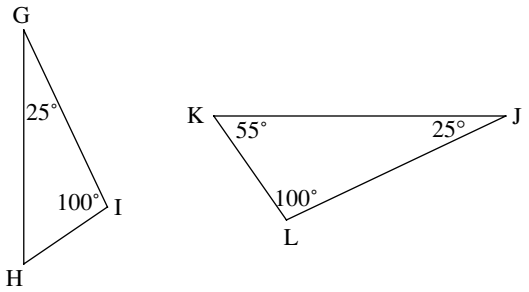


Figure 164

174. Are the triangles in Figure 165 similar?

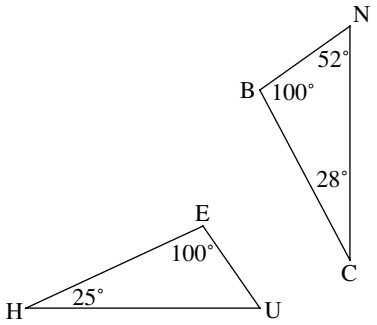


Figure 165

175. Are the triangles in Figure 166 similar?

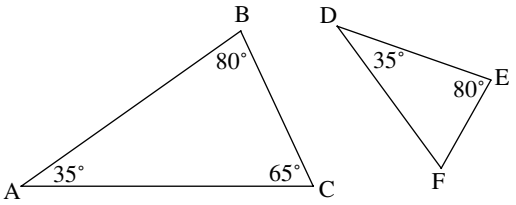


Figure 166

176. Are the triangles in Figure 167 similar?

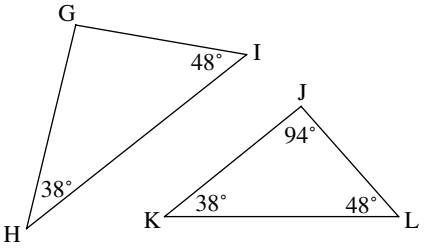


Figure 167

177. Complete this similarity statement for the triangles in Figure 168: ΔIHG ~ Δ_____

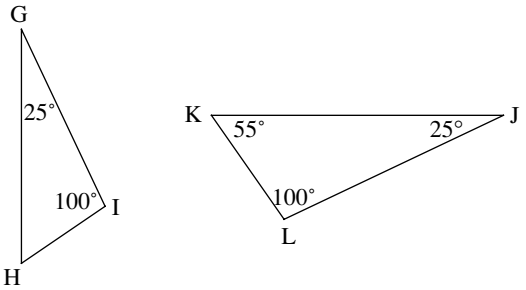


Figure 168

178. Complete this similarity statement for the triangles in Figure 169: ΔBAC ~ Δ_____

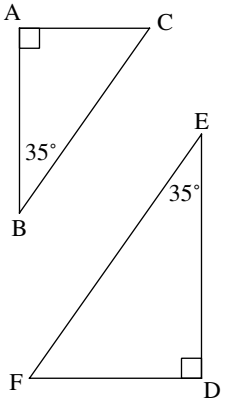


Figure 169

179. Complete this similarity statement for the triangles in Figure 170: ΔTWP ~ Δ_____

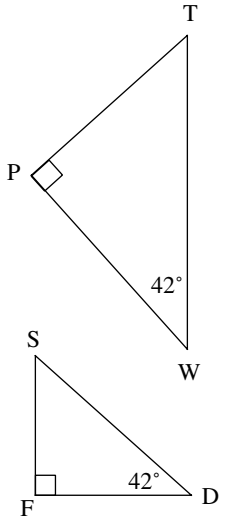


Figure 170

180. Complete this similarity statement for the triangles in Figure 171: ΔCBA ~ Δ_____

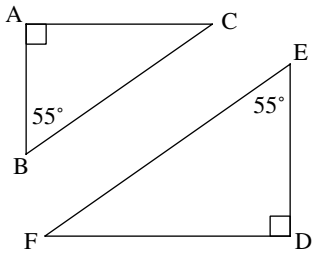


Figure 171

181. Complete this similarity statement for the triangles in Figure 172: ΔBAC ~ Δ_____

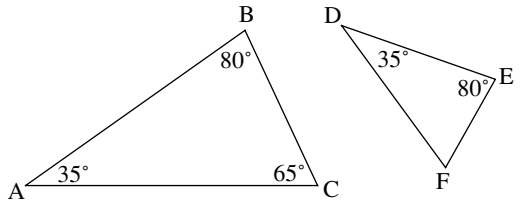


Figure 172

182. Complete this similarity statement for the triangles in Figure 173: ΔGIH ~ Δ_____

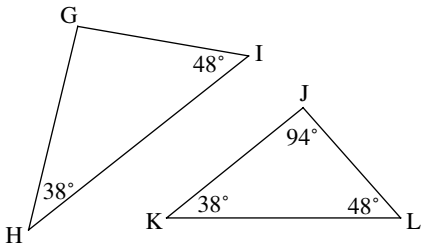


Figure 173

183. Complete this similarity statement for the triangles in Figure 174: ΔABE ~ Δ_____

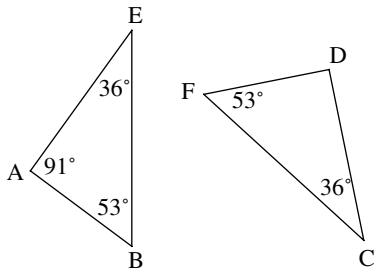


Figure 174

184. Complete this similarity statement for the triangles in Figure 175: ΔFDC ~ Δ_____

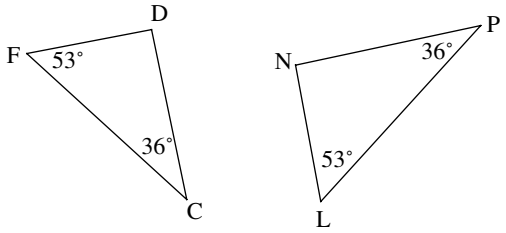


Figure 175

185. Are the triangles in Figure 176 similar?

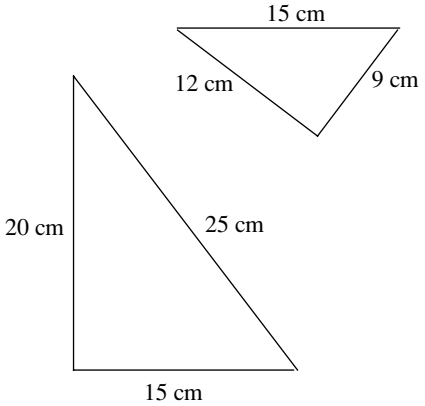


Figure 176

186. Are the triangles in Figure 177 similar?

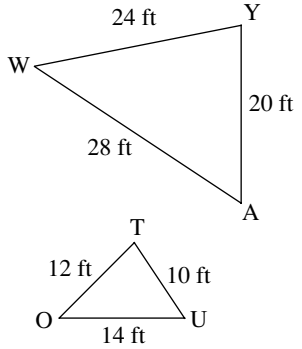


Figure 177

187. Are the triangles in Figure 178 similar?

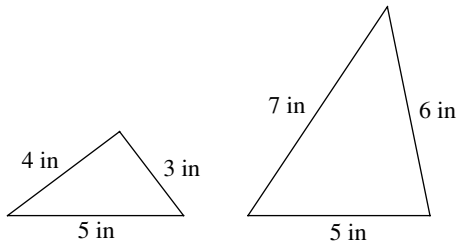


Figure 178

188. Are the triangles in Figure 179 similar?

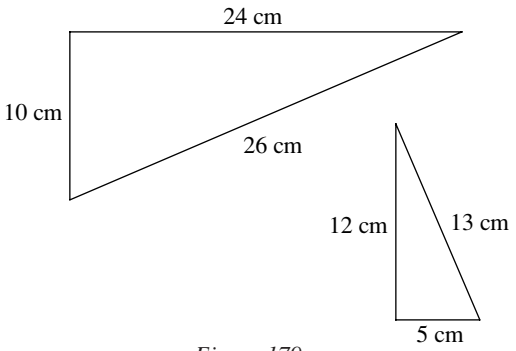


Figure 179

189. Complete this similarity statement for the triangles in Figure 180: ΔAKE ~ Δ_____

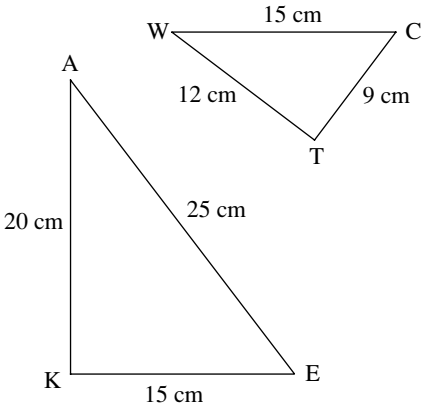


Figure 180

190. Complete this similarity statement for the triangles in Figure 181: ΔWAY ~ Δ_____

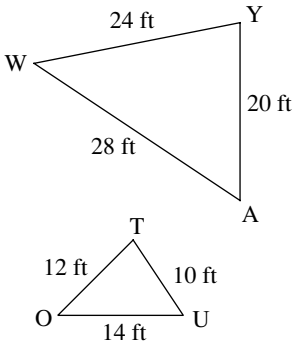


Figure 181

191. Complete this similarity statement for the triangles in Figure 182: ΔCJW ~ Δ_____

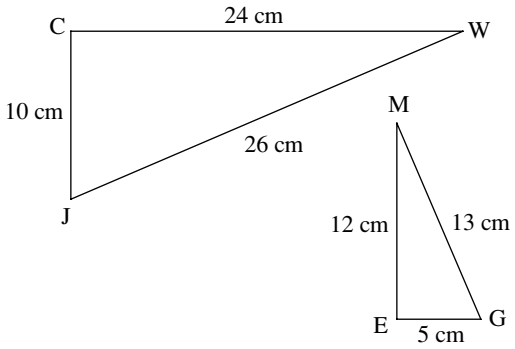


Figure 182

192. Complete this similarity statement for the triangles in Figure 183: ΔBCA ~ Δ_____

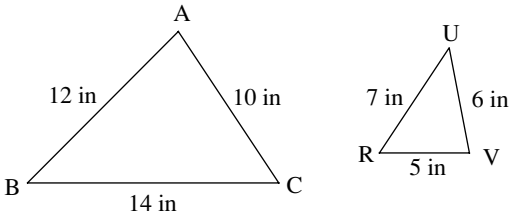


Figure 183

Measurements of Corresponding Angles of Similar Triangles

For help working these types of problems, go back to Example 35 in the Explain section of this lesson.

193. In Figure 184, the triangles are similar. ΔRYE ~ ΔCBV. Find the measure of angle V.

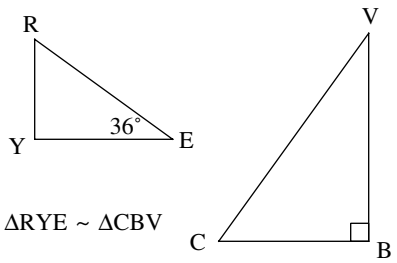


Figure 184

194. In Figure 184, the triangles are similar. ΔRYE ~ ΔCBV. Find the measure of angle Y.

195. In Figure 184, the triangles are similar. ΔRYE ~ ΔCBV. Find the measure of angle C.

196. In Figure 184, the triangles are similar. ΔRYE ~ ΔCBV. Find the measure of angle R.

197. In Figure 185, ΔGHL is similar to ΔTBP. Find the measure of angle B.

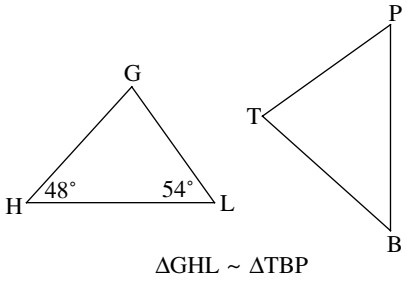


Figure 185

198. In Figure 185, ΔGHL is similar to ΔTBP. Find the measure of angle P.

199. In Figure 185, ΔGHL is similar to ΔTBP. Find the measure of angle G.

200. In Figure 185, ΔGHL is similar to ΔTBP. Find the measure of angle T.

201. In Figure 186, ΔABC is similar to ΔUTV. Find the measure of angle B.

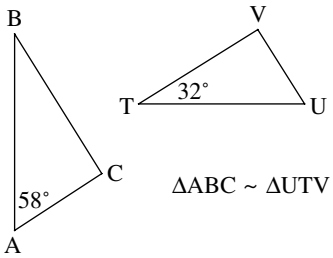


Figure 186

202. In Figure 186, ΔABC is similar to ΔUTV. Find the measure of angle U.

203. In Figure 186, ΔABC is similar to ΔUTV. Find the measure of angle C.

204. In Figure 186, ΔABC is similar to ΔUTV. Find the measure of angle V.

205. In Figure 187, the triangles are similar. ΔBCA ~ ΔRLO. Find the measure of angle B.

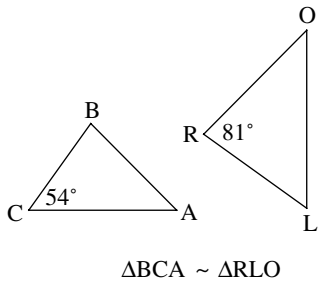


Figure 187

206. In Figure 187, the triangles are similar. ΔBCA ~ ΔRLO. Find the measure of angle A.

207. In Figure 187, the triangles are similar. ΔBCA ~ ΔRLO. Find the measure of angle O.

208. In Figure 187, the triangles are similar. ΔBCA ~ ΔRLO. Find the measure of angle L.

209. In Figure 188, the triangles are similar. ΔABC ~ ΔDEF. Find the measure of of angle B.

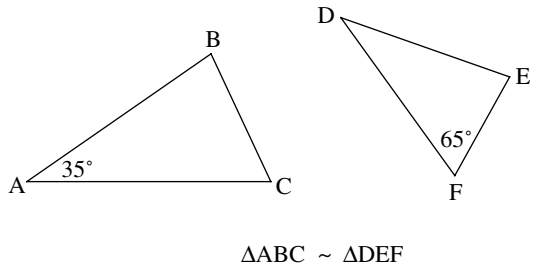


Figure 188

210. In Figure 188, the triangles are similar. ΔABC ~ ΔDEF. Find the measure of angle C.

211. In Figure 188, the triangles are similar. ΔABC ~ ΔDEF. Find the measure of angle D.

212. In Figure 188, the triangles are similar. ΔABC ~ ΔDEF. Find the measure of angle E.

213. In Figure 189, the triangles are similar. ΔGHI ~ ΔJKL. Find the measure of angle G.

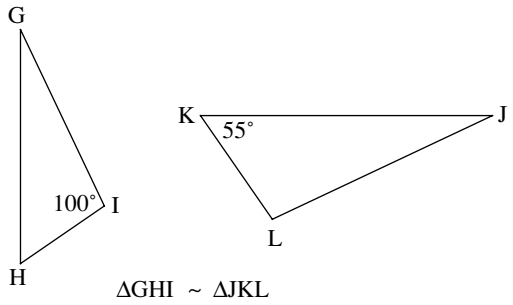


Figure 189

214. In Figure 189, the triangles are similar. ΔGHI ~ ΔJKL. Find the measure of angle H.

215. In Figure 189, the triangles are similar. ΔGHI ~ ΔJKL. Find the measure of angle J.

216. In Figure 189, the triangles are similar. ΔGHI ~ ΔJKL. Find the measure of angle L.

Lengths of Corresponding Sides of Similar Triangles

For help working these types of problems, go back to Examples 36–38 in the Explain section of this lesson.

217. The triangles in Figure 190 are similar. ΔWAY ~ ΔOUT. Find x, the length of $\overline{UT}$.

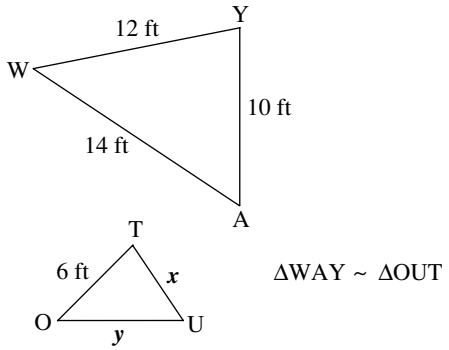


Figure 190

218. The triangles in Figure 190 are similar. ΔWAY ~ ΔOUT. Find y, the length of $\overline{OU}$.

219. The triangles in Figure 191 are similar. ΔCJW ~ ΔEGM. Find x, the length of $\overline{CJ}$.

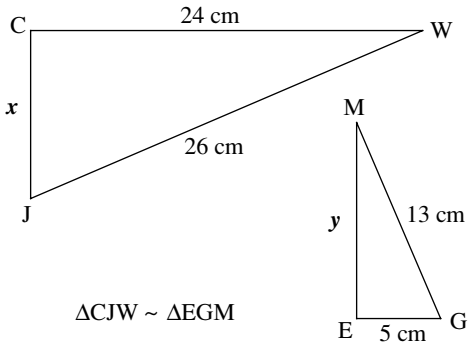


Figure 191

220. The triangles in Figure 191 are similar. ΔCJW ~ ΔEGM. Find y, the length of $\overline{EM}$.

221. The triangles in Figure 192 are similar. ΔAKE ~ ΔWTC. Find x, the length of $\overline{AK}$.

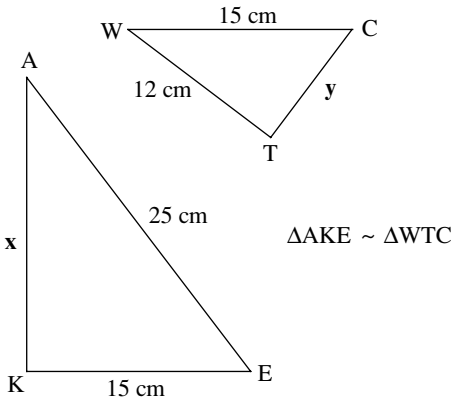


Figure 192

222. The triangles in Figure 192 are similar. ΔAKE ~ ΔWTC. Find y, the length of $\overline{TC}$.

223. The triangles in Figure 193 are similar. ΔBKR ~ ΔDPC. Find x, the length of $\overline{PC}$.

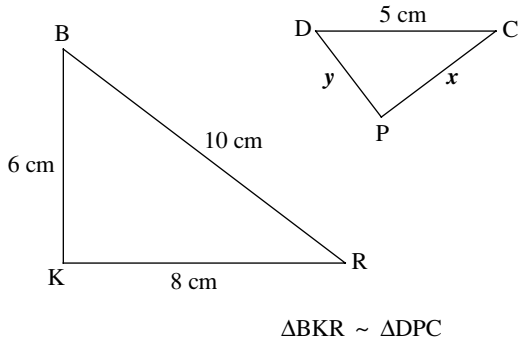


Figure 193

224. The triangles in Figure 193 are similar. ΔBKR ~ ΔDPC. Find y, the length of $\overline{DP}$.

225. The triangles in Figure 194 are similar. ΔABC ~ ΔADE. Find x, the length of $\overline{BC}$.

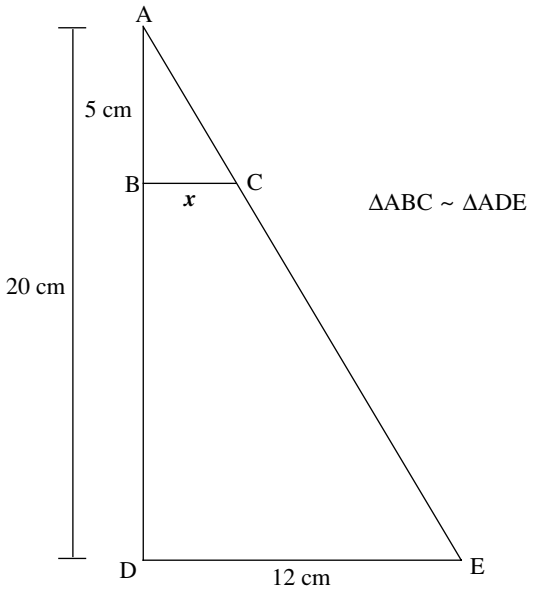


Figure 194

226. The triangles in Figure 195 are similar. ΔJKL ~ ΔJMN. Find x, the length of $\overline{MN}$.

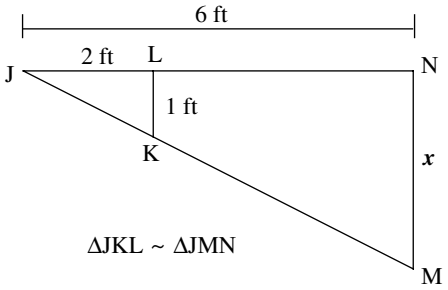


Figure 195

227. The triangles in Figure 196 are similar. ΔABC ~ ΔADE. Find x, the length of $\overline{AB}$.

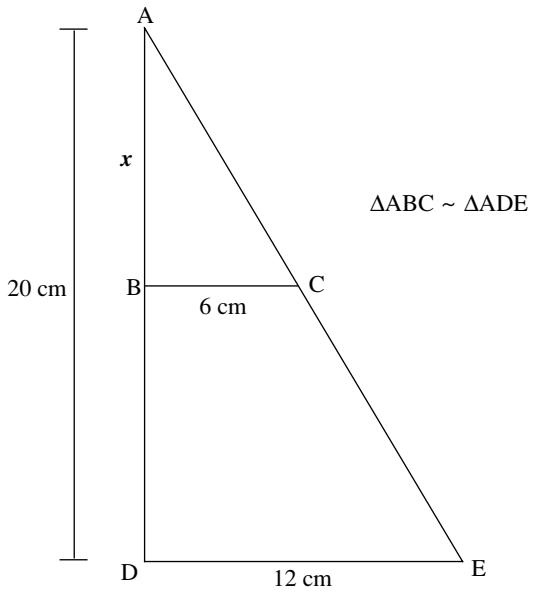


Figure 196

228. The triangles in Figure 197 are similar. ΔJKL ~ ΔJMN. Find x, the length of $\overline{KL}$.

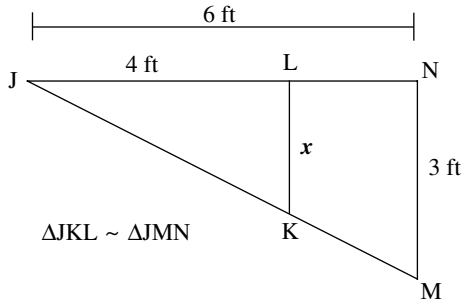


Figure 197

229. The triangles in Figure 198 are similar. ΔABC ~ ΔADE. Find x, the length of $\overline{BC}$.

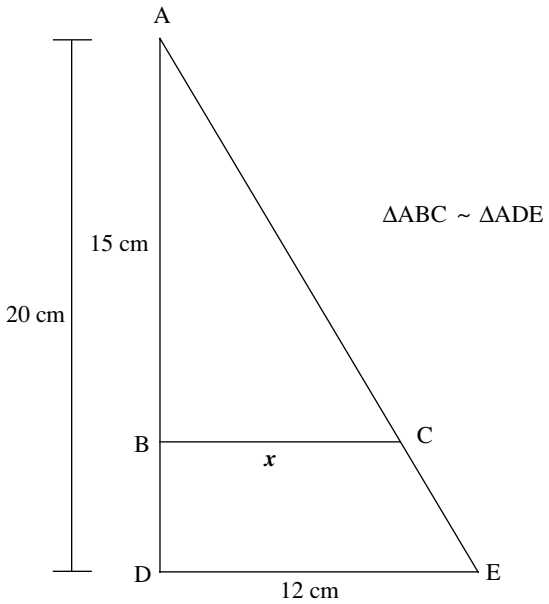


Figure 198

230. The triangles in Figure 199 are similar. ΔPQR ~ ΔPST. Find x, the length of $\overline{QR}$.

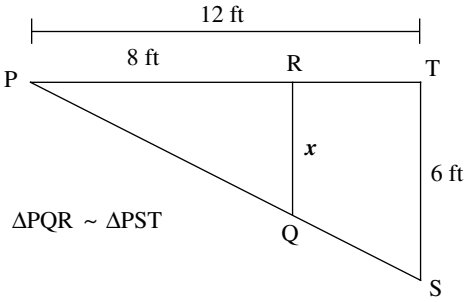


Figure 199

231. The triangles in Figure 200 are similar. ΔPQR ~ ΔPST. Find x, the length of $\overline{PR}$.

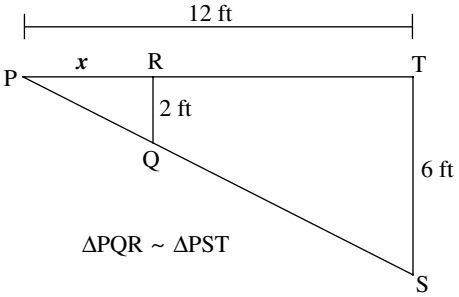


Figure 200

232. The triangles in Figure 201 are similar. ΔTUV ~ ΔTMN. Find x, the length of $\overline{UT}$.

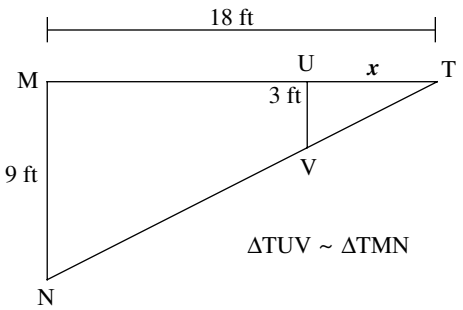


Figure 201

233. The triangles in Figure 202 are similar. ΔABC ~ ΔADE. Find x, the length of $\overline{BD}$.

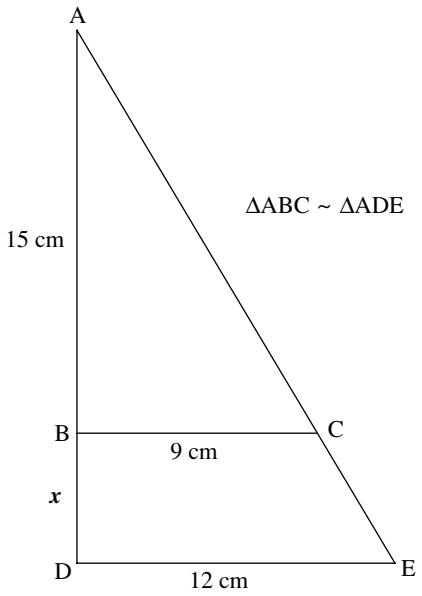


Figure 202

234. The triangles in Figure 203 are similar. ΔFGH ~ ΔFIJ. Find x, the length of $\overline{GI}$.

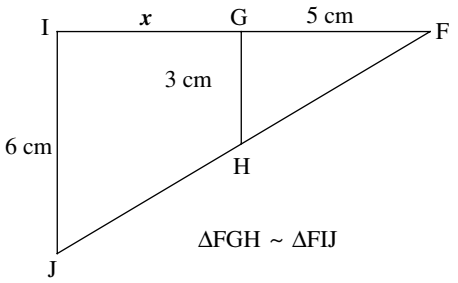


Figure 203

235. The triangles in Figure 204 are similar. ΔPQR ~ ΔPST. Find x, the length of $\overline{RT}$.

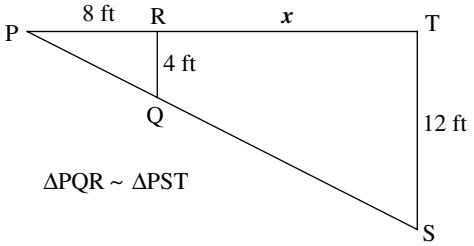


Figure 204

236. The triangles in Figure 205 are similar. ΔPQR ~ ΔPST. Find x, the length of $\overline{RT}$.

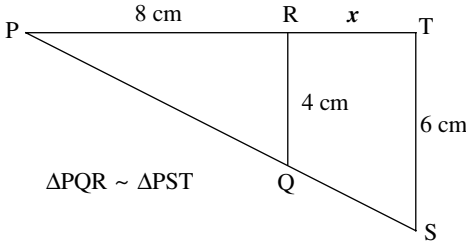


Figure 205

237. The triangles in Figure 206 are similar. ΔJKL ~ ΔJMN. Find x, the length of $\overline{LJ}$.

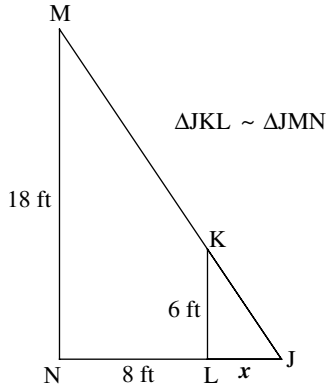


Figure 206

238. The triangles in Figure 207 are similar. ΔPEA ~ ΔPOD. Find x, the length of EO.

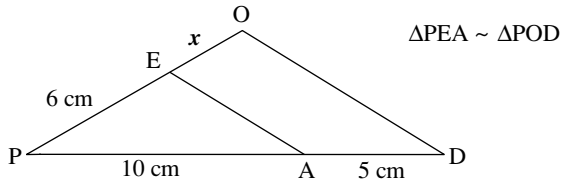


Figure 207

239. The triangles in Figure 208 are similar. ΔJKL ~ ΔJMN. Find x, the length of NL.

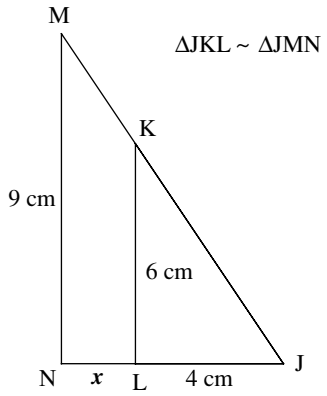


Figure 208

240. The triangles in Figure 209 are similar. ΔPEA ~ ΔPOD. Find x, the length of PA.

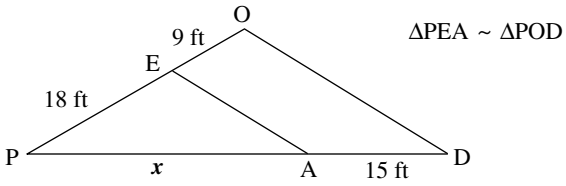


Figure 209

Evaluate

Take this Practice Test to prepare for the final quiz in the Evaluate module of this lesson on the computer.

Practice Test

1. One of the angles in a right triangle measures 10°. Find the measure of the other acute angle of the triangle.

2. In Figure 210, ΔABC is congruent to ΔDEF.
   length $\overline{AB}$ = length $\overline{BC}$
   m∠A = 71°
   Find the measure of ∠E.

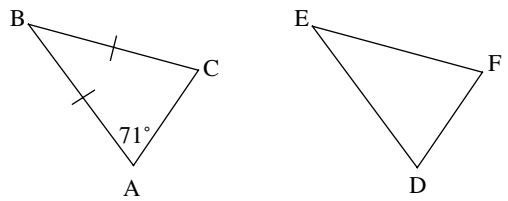


Figure 210

3. In Figure 211:
   ΔJKL is an equilateral triangle.
   length $\overline{LK}$ = 24 cm
   $h = 12\sqrt{3}$ cm
   Find x, the length of $\overline{JM}$.

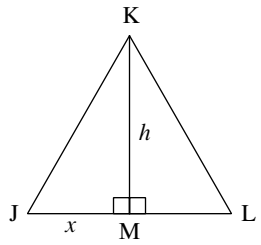


Figure 211

4. In Figure 212:
line $\overline{AB}$ is parallel to line $\overline{CD}$
line $\overline{AC}$ is parallel to line $\overline{BD}$
length $\overline{AB}$ = 8.1 cm
length $\overline{AC}$ = 10.2 cm
m∠ACD = 54°
Find length $\overline{BD}$, m∠ABD, and m∠BDC.

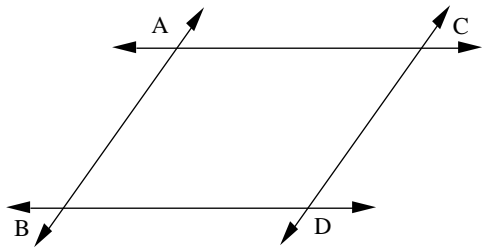


Figure 212

5. The length of each side of a given pentagon measures 2 in. A larger pentagon which is similar to the given pentagon has a side that measures 2.4 in. Find the perimeter of the larger pentagon.

6. Triangle ABC is similar to triangle DEF.
AB = 14 cm BC = 15 cm AC = 8 cm DE = 42 cm
Find x, the length of $\overline{EF}$. (Hint: Sketch the two triangles, making sure to match corresponding sides.)

7. In Figure 213:
$\overline{AB}$ is parallel to $\overline{DE}$
AB = 130 cm
BC = 117 cm
DE = 390 cm
Find y, the length of $\overline{DC}$.

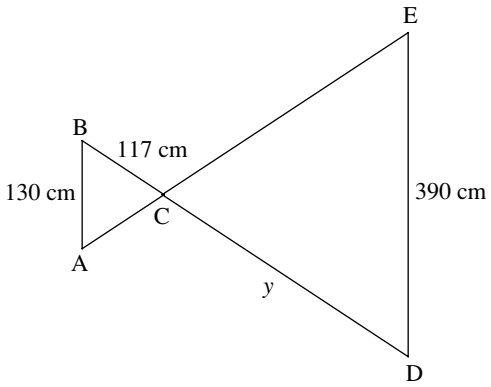


Figure 213

8. In Figure 214:
$\Delta CED \sim \Delta AEB$
EC = 21 cm
CD = 18.9 cm
AB = 37.8 cm
Find x, the length of $\overline{EA}$.

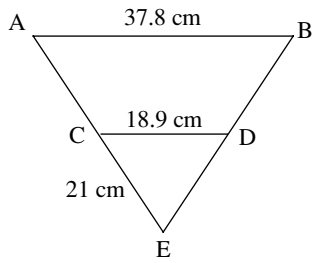


Figure 214

Topic F5 Cumulative Review

These problems cover the material from this and previous topics. You may wish to do these problems to check your understanding of the material before you move on to the next topic, or to review for a test.

1. Find: $180° - (48\frac{1}{2}° + 22\frac{1}{4}°)$

2. In Figure F5.1, find the measure of ∠F.

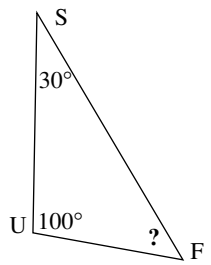


Figure F5.1

3. In Figure F5.2, find the measure of angle A.
(Hint: ΔPLA is an equilateral triangle.)

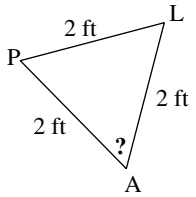


Figure F5.2

4. Find: 22.5 cm + 10.2 cm + 22.5 cm + 10.2 cm

5. In Figure F5.3, name a point, a line segment, a ray, and a line.

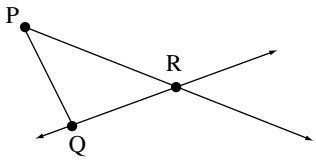


Figure F5.3

6. Use the order of operations to do this calculation: $\frac{1}{3} \cdot (4.4 + 4.6) + 4$

7. Melissa is building a fence around her rectangular garden. The garden is 16 yards long and 10 yards wide. How many yards of fence must she build in order to completely enclose the garden?

8. Find: $180° - (32.4° + 27.8°)$

9. Name the polygon that has 8 sides.

10. Maurice is painting the ceiling in his living room. The ceiling is $5\frac{1}{3}$ yards by 6 yards. How many square yards will he have to paint to give the entire ceiling one coat of paint?

11. Find the area of the parallelogram shown in Figure F5.4.

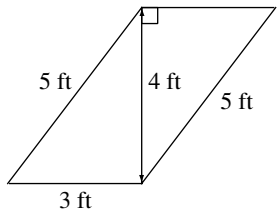


Figure F5.4

12. Give the degree measure of a right angle.

13. If an angle measures 100.2°, is it an acute, obtuse, right, or straight angle?

14. In Figure F5.6, find the length of $\overline{ZX}$. (Hint: ΔXYZ is an isosceles triangle.)

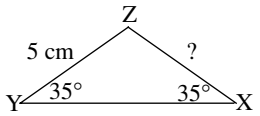


Figure F5.6

15. The most popular item of a pennant manfacturer is the classic triangular felt pennant. If this pennant is 16.5 inches long from the center of its base to its tip and its base is 8 inches long, how many square inches of felt are needed to make the pennant?

16. In Figure F5.7, **f** ∥ **g**. The measure of angle 8 is 75°. Find the measure of angle 2.

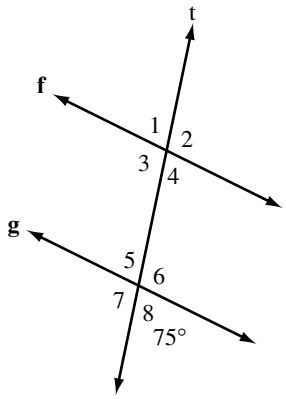


Figure F5.7

17. In Figure F5.8, find the measure of ∠Y.

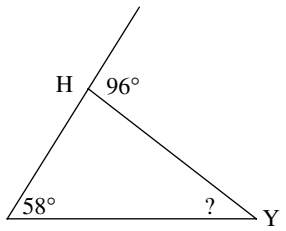


Figure F5.8

18. In a right triangle, the measure of one angle is 62°. Find the measures of the other angles.

19. The quadrilateral in Figure F5.9 is a parallelogram. Find the measure of angle L.

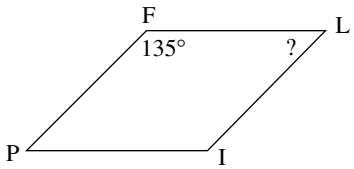


Figure F5.9

20. If an angle measures $33\frac{1}{3}°$, is it an acute, obtuse, right, or straight angle?

21. Find the area of the trapezoid shown in Figure F5.10.

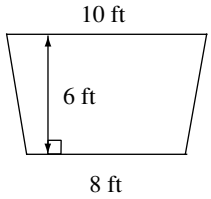


Figure F5.10

22. If m∠U = 82.5° and m∠V = 97.5°, are angles U and V complementary, supplementary, or neither?

23. The radius of a circle is 18 inches. Find its circumference and its area.

24. In Figure F5.11, what is the measure of ∠BAC?

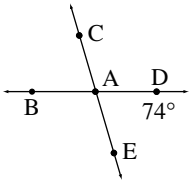


Figure F5.11

25. ΔABC is congruent to ΔDEF. Which angle of ΔDEF is congruent to ∠F?

26. The lengths of the sides of a certain right triangle are 80 cm, 84 cm, and 116 cm. Find the length of the hypotenuse.

27. Bud is covering the surface of a gift box with gold foil. The box is a rectangular prism that is 4 inches tall, 5 inches wide, and 8 inches long. How many square inches of foil are needed to cover the entire surface of the box with no overlap?

28. Al is building a trunk to store bedding and other items. He figures that a trunk with a volume of 9 cubic feet should be large enough. If he builds the trunk so that it is 2 feet high and 3 feet long, how wide should he make the trunk so that it has a volume of 9 cubic feet?

29. ΔABC is congruent to ΔDEF. Which side of ΔDEF is congruent to $\overline{BC}$?

30. The radius of a cylinder is 2 inches. The height of the cylinder is 6 inches. What is the surface area of the cylinder?

31. The radius of a cylinder is 2 inches. The height of the cylinder is 6 inches. What is the volume of the cylinder? Use 3.14 to approximate π. Round your answer to two decimal places.

32. Find the value of this expression: –122 – 82

33. The lengths of the sides of ΔUSA are 9 ft, 10 ft, and 12 ft. Is ΔUSA a right triangle?

34. The diameter of the base of a cone is 3 centimeters. The height of the cone is 12 centimeters. What is the volume of the cone? Use 3.14 to approximate π. Round your answer to two decimal places.

35. Are the polygons in Figure F5.12 similar polygons?

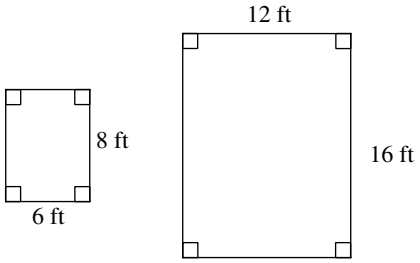


Figure F5.12

36. ΔFAR ~ ΔOUT. (Triangle FAR is similar to triangle OUT.) Find the angle in ΔOUT that has the same measure as angle A.

37. ΔFAR ~ ΔOUT. The ratio of FA to OU is 8 to 3. Find the ratio of AR to UT.

38. Complete this similarity statement for the triangles in Figure F5.13:
ΔSRC ~ Δ_____

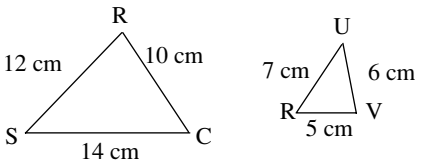


Figure F5.13

39. The triangles in Figure F5.14 are similar. ΔGHI ~ ΔJKL. Find the measure of angle G.

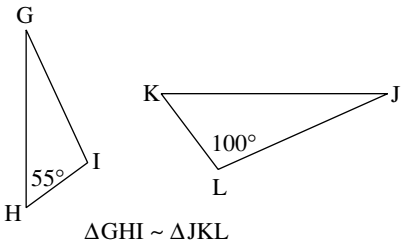


Figure F5.14

40. Find the missing number, x, that makes this proportion true: $\frac{2}{5} = \frac{12}{x}$.

41. The triangles in Figure F5.15 are similar. ΔPQR ~ ΔPST. Find x, the length of PR.

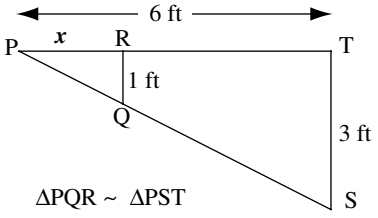


Figure F5.15

42. The triangles in Figure. F5.16 are similar. $\Delta JKL \sim \Delta JMN$ Find x, the length of $\overline{NL}$.

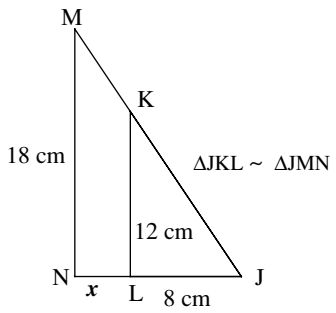


Figure F5.16

43. The volume of a sphere is 36π cubic feet. What is the radius of the sphere?